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Functional law of the iterated logarithm for lacunary trigonometric and some gap series

By

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0. Introduction

The purpose of this paper is to prove the functional law of the iterated logarithm for lacunary trigonometric series and some gap series under probability measures with some regularity.

Let us first recall two limit theorems for a sequence of i.i.d. random variables.

Theorem A. *Let* $\{\xi_i\}$ *be a sequence of i.i.d. with mean* 0 *and variance* 1 *and put* $S_n=\xi_1+\cdots+\xi_n$. *Then it holds that*

$$\limsup_{n\to\infty}\frac{S_n}{\sqrt{2n\log\log n}}=1 \quad a.s.$$

Theorem B. *Let* $\{\xi_i\}$ *satisfy the same conditions as those of Theorem A and let us denote by* $\{X_n\}$ *a sequence of* $C[0, 1]$ *valued random variables such that* $X_n(s)=(S_{[ns]}+(ns-[ns])\xi_{[ns]+1})/\sqrt{n}$. *Then the sequence* $\{X_n/\sqrt{2\log\log n}\}$ *is relatively compact in* $C[0, 1]$ *and the set of all clusters of this sequence coincides with* K, *almost surely, where* $K=\{x\in C[0, 1]; x(0)=0,\ x$ *is absolutely continuous and* $\int_0^1\dot{x}^2(t)dt\leqq 1\}$.

Theorem A is the classical version of the law of the iterated logarithm (LIL) due to Hartman-Wintner [9] and Theorem B is its functional version due to Strassen [23] which is called Strassen's law of the iterated logarithm or functional law of the iterated logarithm (FLIL). There are various extensions of these results to the case of dependent sequence of random variables, for example, mixing sequences, martingale difference sequences, lacunary trigonometric series, some gap series of functions and multiplicative systems. We shall here state two results due to Takahashi on lacunary trigonometric series corresponding to Theorem A and B resprctively. In the following two theorems, lacunary trigonometric series are regarded as random series on the probability space $([0, 1], dx)$.

Theorem C. *Suppose that sequences* $\{n_j\}$ *of integers and* $\{a_j\}$ *of real numbers satisfy following conditions.*

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