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The Bers projection and the λ-lemma

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§ 0. Introduction

In order to discuss the connection between univalent functions and Teichmüller spaces, it is important to consider the class S of Schwarzians of all schlicht functions on the exterior Δ^* of the unit disk, i.e.,

$$S=\left\{S_f=(f''/f')'-\frac{1}{2}(f''/f')^2 : f\in\Sigma_0\right\},$$

where Σ_0 is the class of all univalent meromorphic functions f on Δ^* having an expansion

$$f(z)=z+\sum_{n=1}^{\infty} b_n z^{-n} .$$

It should be noted that the correspondence $f\mapsto S_f$ is a bijection from Σ_0 to S.

The class S inherits a topology by the hyperbolic sup-norm of weight -2 (so-called the Nehari norm) of the space of holomorphic quadratic differentials. The space S has been studied by many authors (Bers [6], Gehring [15], [16], Žuravlev [28], Flinn [13], Shiga [24], Overholt [20], Sugawa [25], etc.). In particular, the first remarkable result by Gehring [15] states that

$$\operatorname{Int} S=T(=\text{the universal Teichmüller space}).$$

As the Bers projection plays a very important role in the Teichmüller theory, the (generalized) Bers projection is thought to do so in the investigation of the space S, too. § 1 is devoted to study the (generalized) Bers projection mainly in the case that the domain has no exterior (Theorem 1). As a corollary of Theorem 1, we give a simple proof of a theorem of Overholt [19].

In § 2 and succesive sections, we shall consider $\operatorname{Int} S(\Gamma)$, where Γ is an arbitrary Fuchsian group. The "λ-lemma" and the "improved λ-lemma" first introduced by Mañé-Sad-Sullivan [17] and Sullivan-Thurston [26] are greatly powerful tools to study the structure of holomorphic families and, indeed, have many applications in various aspects (for example, see [7], [9], [11], [21], [24]).

As a new application of the "extended λ-lemma" (Bers-Royden [9]), in § 2, we give another proof of a theorem of Žuravlev: $T(\Gamma)$ is the zero component of $\operatorname{Int} S(\Gamma)$. Our proof is based only on the openness of the universal Teichmüller space due to Ahlfors and the λ-lemma while Žuravlev's one is essentially relies upon the Grunsky's in-

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