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Ergodic properties of discrete groups; inheritance to normal subgroups and invariance under quasiconformal deformations

By

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§0. Introduction

There are many studies on the ergodic properties of discrete isometry groups Γ acting on the n-dimensional hyperbolic space $B^n = \{x \in \mathbf{R}^m; |x| < 1\}$ and on the sphere at infinity $S^{n-1} = \{x \in \mathbf{R}^n; |x| = 1\}$ (cf. [N]). Among them, Lyons and Sullivan's work [LS] is remarkable. They obtained the conditions concerning covering transformation groups, under which normal (regular) covers of a compact hyperbolic manifold are recurrent or Liouville. In other words, we may say that they showed what normal subgroups inherit the ergodicity of the action on S^{n-1} with respect to the Lebesgue measure from a cocompact discrete group. In connection with this problem, in the present paper, we consider in what degree any normal subgroup Γ' of Γ inherits ergodicity on $S^{n-1} \times S^{n-1}$ ($= B^n/\Gamma$ is recurrent) and ergodicity on S^{n-1} ($= B^n/\Gamma$ is Liouville). Particularly, in the case where $n = 2$, we can characterize $S^{n-1} \times S^{n-1}$-ergodicity of Γ by conservativity of the action on S^{n-1} of Γ':

Theorem. *A Riemann surface B^2/Γ is recurrent if and only if any non-trivial normal subgroup of Γ is conservative.*

We develop those arguments in the first part "inheritance to normal subgroups" (§4 and §5) after the sections of several preliminaries and preparations. The first part also contains some investigations on the following two conjectures which seem interesting in the course of our arguments:

(C1) If B^n/Γ is recurrent and Γ' is a normal subgroup of Γ such that any subgroup of Γ/Γ' is a finitely generated solvable group, then B^n/Γ' is Liouville (cf. [LS]).

(C2) Γ acts on S^{n-1} ergodically if and only if any normal subgroup Γ' of Γ acts on S^{n-1} either conservatively or totally dissipatively.

In the second part "invariance under quasiconformal deformations" (§6 and §7), we study whether the ergodic properties on S^{n-1} are preserved or not, by

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