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Some prime ideals in the extensions of Noetherian rings

By

Junro SATO

Introduction

Let R and S be commutative rings with unity and let $f\colon R \to S$ be a homomorphism with $f(1) = 1$. Mcquillan, in his paper [2], introduces the notions of S-prime and S-primary ideals. In fact, he proved that if f is a flat homomorphism, then every prime ideal of S is S-prime, and that if R is integrally closed integral domain, S is integral over R and no non-zero element of R is a divisor of zero in S, then every prime ideal of R is S-primary. The aim of this paper is to investigate more closely S-prime and S-primary ideals. Further, we introduce the notion of S-quasi-primary, and determine its structure. We are mainly interested in the case where R and S are Noetherian. If $f\colon R \to S$ is a homomorphism of rings and if I is an ideal of S, then $f^{-1}(I)$ is denoted by $I \cap R$.

In the first section, we consider S-prime ideals. In fact, we shall prove that $\mathfrak{a}$ is S-prime if and only if $\mathfrak{a}$ is a prime ideal of R and $\mathfrak{a}S \cap R = \mathfrak{a}$, provided that $\mathfrak{a}S \neq S$.

In the second section, we discuss S-primary ideals.

In the final section, we introduce and study S-quasi-primary ideals. The main result is that $\mathfrak{a}$ is an S-quasi-primary if and only if $\mathfrak{a}S \cap R$ is an R-primary ideal of R such that $\sqrt{\mathfrak{a}} = \sqrt{\mathfrak{a}S \cap R} = p$, and that $\mathrm{Ass}_R(S/\mathfrak{a}S) = \{p\}$.

In this article R and S are assumed to be commutative rings and to have unity unless otherwise specified, and $f\colon R \to S$ is a homomorphism with $f(1) = 1$, and that our general references for unexplained technical terms are [1] and [3].

§ 1. S-prime ideals

First, we recall the following definition.

Definition 1.1. Let $f\colon R \to S$ be a homomorphism and let q be an ideal of R. We say that q is S-prime if, $a \in R$, $\alpha \in S$ and $f(a)\alpha \in qS$, implies $a \in q$, or $\alpha \in qS$.

Remark 1.2. Let q be an ideal of R such that $qS = S$. Then q is S-prime.

Now, we show our key lemma.

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