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BMO extension theorem for relative uniform domains

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§ 1. Introduction

Let D be a domain in n-dimensional Euclidean space, and $BMO(D)$ the space of all functions of (n-dimensional) bounded mean oscillation on D. We say D has BMO extension property if each $BMO(D)$ function is the restriction to D of some $BMO(\mathbf{R}^n)$ function.

In 1980, P. Jones [J] showed that a domain D has BMO extension property if and only if D is a uniform domain (cf. [GO]). For various characterizations of uniform domain, see [G]. A uniform domain is 'uniform' as a subdomain of $\mathbf{R}^n$ or $\mathbf{R}^n \cup \{\infty\}$. Here we consider relative uniformness of domains, that is, a uniformness as a subdomain of other domain, and show that this relative uniformness and the corresponding relative BMO extension property coincides with to each other, which is a generalization of Jones' result (Th. 1.)

Our method is essentially almost the same as the original one of Jones, but since we must localize his method, and for the completeness, we shall give the proofs for all our lemmas below.

§ 2. Notation, preliminary lemmas and main result

Throughout this paper we treat only 2-dimensional case for the simplicity, since the same argument holds in the case of general dimension. Let D be a domain lying in $\mathbf{R}^2$. We say that a function $u \in L^1_{loc}(D)$ is in $BMO(D)$ if

$$\|u\|_{*,D} = \sup_Q \frac{1}{m(Q)} \int_Q |u(z) - u_Q|\, dm(z) < \infty ,$$

where dm is the two dimensonal Lebesgue measure, $u_Q = m(Q)^{-1} \int_Q u\,dm$ and the supremum is taken for every closed square Q in D whose sides are parallel to the coodinate axes. Throughout this paper 'square' means a closed square whose sides are parallel to the coordinate axes, 'dyadic square' means a square $[k2^n, (k+1)2^n] \times [l2^n, (l+1)2^n]$, k, l, $n \in \mathbf{Z}$, $l(Q)$ denotes the side length of a square Q, tQ, $t > 0$ denotes the square having the same center as Q and $tl(Q)$ as its side length, $d(\cdot, \cdot)$ denotes the Euclidean distance, A_1, A_2, ... denotes

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