

Free complexes defining maximal quasi-Buchsbaum graded modules over polynomial rings

By

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Introduction

In the study of homogeneous ideals defining two codimensional locally Cohen-Macaulay subschemes of a projective space, any knowledge of maximal generalized Cohen-Macaulay graded modules over a polynomial ring R (i.e. graded modules M of the same Krull dimension as that of the ground ring R such that $l_R(H_m^i(M)) < \infty$ for all $i < \dim(R)$, m denoting the irrelevant maximal ideal of R) is very useful in two respects. First, for a homogeneous ideal α of height two in a polynomial ring $R = k[x_1, \dots, x_r]$ having the property $l_R(H_m^i(R/\alpha)) < \infty$ for $i < r - 2$, there is an exact sequence

$$0 \rightarrow N \xrightarrow{\tau} M \rightarrow \alpha \rightarrow 0$$

with a maximal generalized Cohen-Macaulay graded R -module M and a graded free R -module N such that $H_m^{r-1}(M) = 0$, so that once the structure of M , such as its syzygies, is fully understood, the problem can be reduced to the analysis of the linear mapping τ . Second, denoting by t_0 the minimum of all t such that $\alpha_t \neq 0$, let M' be the module over $R' = k[x_2, \dots, x_r]$ defined by the exact sequence

$$0 \rightarrow M' \rightarrow \bigoplus_{i=0}^{t_0-1} R'(-i) \xrightarrow{(1, x_1^1, \dots, x_1^{i_0-1})} R/\alpha \rightarrow 0,$$

where the linear forms x_1, \dots, x_r are chosen sufficiently generally. Then $H_m^i(M') \cong H_m^{i-1}(R/\alpha)$ for each $i < r - 1$ as an R' -module, so M' is a maximal generalized Cohen-Macaulay R' -module satisfying $e_{m'}(M') = t_0$, which bears a lot of information on the generators of α . In fact, applying Goto's structure theorem for maximal Buchsbaum modules over regular local rings (see [G2, (3.1)], [EG, Theorem 3.2]) to the above M, M' , we could give a complete classification of homogeneous prime ideals that define arithmetically Buchsbaum subvarieties of codimension two in projective spaces (see [A1, §7]).

Keeping that in mind, we will investigate the structure of maximal quasi-Buchsbaum graded modules over polynomial rings (i.e. graded modules M over R with $mH_m^i(M) = 0$ for all $i < \dim(R)$), especially in a simple case where $\iota(M) :=$