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Charge transfer model and (2-cluster) → (2-cluster) three-body scattering

By

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§ 1. Introduction

We consider a three-body system consisting of two heavy particles (particles 1, 2) with the masses M_1, M_2 and a light particle (particle 3) with m. We set $\mu = (M_1, M_2)$ and write $\mu \gg 1$ ($\mu \to \infty$) for M_1, $M_2 \gg 1$ ($M_1, M_2 \to \infty$). Let $r_j \in \mathbf{R}^N$ ($j = 1, 2, 3$), $N \geqq 2$, be the position of particle j, and let V_{jk} be the pair potential between particle j and particle k. Then the three-body Hamiltonian is

$$(1.1)\qquad \begin{aligned} &\tilde{H}^\mu = -\sum_{j=1}^{2} (2M_j)^{-1}\Delta_{r_j} - (2m)^{-1}\Delta_{r_3} + V \qquad \text{in } L^2(\mathbf{R}^{3N}), \\ &V = V(r_1, r_2, r_3) = V_{23}(r_3 - r_2) + V_{13}(r_3 - r_1) + V_{12}(r_2 - r_1). \end{aligned}$$

We assume the following throughout this paper:

(V) $V_{ij}(x)$ $(1 \leqq i < j \leqq 3)$ *is a smooth real-valued function on* $\mathbf{R}^N$, *and there exists* $\varepsilon_0 > N + (3/2)$ *such that*

$$|\partial_x^\gamma V_{ij}(x)| \leqq C_\gamma (1 + |x|)^{-\varepsilon_0}$$

for all multi-indices γ.

Our main results are Theorems 1.1 and 1.3, which will be stated at the end of this section. For the proof of Theorem 1.1, we assume further

(V)′ $V_{ij}(x)$ $(1 \leqq i < j \leqq 3)$ *satisfies* (V) *with*

$$\varepsilon_0 > [(N-1)/2] + N + (3/2). \qquad ([\quad] \textit{ is Gauss' symbol.})$$

As usual, we remove the kinetic energy of the center of mass from $\tilde{H}^\mu$ to get an operator H^μ in $L^2(\mathbf{R}^{2N})$. A 2-cluster decomposition of the set $\{1, 2, 3\}$ is a partition of $\{1, 2, 3\}$ into two nonempty subsets, and in particular we use only the following 2-cluster decompositions:

$$(1.2)\qquad a_1 := \{1, (2, 3)\}, \qquad a_2 := \{2, (1, 3)\},$$

and we define $\mathbf{A} := \{a_1, a_2\}$.

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