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Singular Cauchy Problems of Higher Order with Characteristic Initial Surface

By

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Introduction

The present article is concerned with the Cauchy problem of linear partial differential equation with holomorphic coefficients in complex domain. The purpose is to give an explicit representation of the singularity of the solution for meromorphic Cauchy data.

In the case where the initial surface is non-characteristic, this problem has been studied by several authors: see Y. Hamada [2] in case of simple characteristics, see Y. Hamada–J. Leray–C. Wagschal [3] in case of constant multiple characteristics, see Y. Hamada–G. Nakamura [4], D. Shiltz–J. Vaillant–C. Wagschal [10] and T. Kobayashi [8] in case of involutive characteristics, and see, for instance, J. Urabe [12] and C. Wagschal [14] and so on in other cases.

On the other hand, we can consider this problem even in the case where the initial surface is characteristic. Indeed, the Cauchy problem for Fuchsian partial differential operator (in the sense of M. S. Baouendi–C. Goulaouic [1]) has a unique holomorphic local solution under some conditions (see Y. Hasegawa [5], M. S. Baouendi–C. Goulaouic [1]). J. Urabe [13] treated a special class of operaters in $\mathbf{C}^2$ whose principal parts are $t\partial_t^2 - \partial_x^2$ and whose characteristic exponents are constant. He gave an explicit representation of the singularity of the solutions by means of hypergeometric functions. S. Ouchi [8] treated second order operators whose principal parts are of simple characteristics multiplied by t^2. He used the multi-phase functions and showed that the solutions are holomorphic except on the characteristic sets.

In this paper, we treat a class of operators $L(x; D_x)$, $x = (x_0, x_1, \dots, x_n) = (x_0, x')$ of order $2m$ ($m \in \mathbf{N}$), which are, roughly speaking, transformed to operators with simple characteristics by change of variables $x_0 = y_0^2$, $x' = y'$ (see (A.1) and (A.2)). By (A.1), these are of Fuchs type with weight m. So we consider the Cauchy problem

$$\begin{cases} L(x; D_x)u(x) = 0, \\ D_0^k u(0; x') = v_k(x') \qquad (k = 0, \dots, m-1), \end{cases}$$

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