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The K_*-localizations of the stunted real projective spaces

By

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0. Introduction

Given an associative ring spectrum E with unit, a CW-spectrum X is said to be *quasi E_*-equivalent* to a CW-spectrum Y if there exists a map $f: Y \to E \wedge X$ such that the composite $(\mu \wedge 1)(1 \wedge f): E \wedge Y \to E \wedge X$ is an equivalence where $\mu: E \wedge E \to E$ denotes the multiplication of E. We call such a map $f: Y \to E \wedge X$ a quasi E_*-equivalence. Let KO and KU be the real and the complex K-spectrum respectively. Since there is no difference between the KO_*- and KU_*-localizations, we denote by S_K the K_*-localization of the sphere spectrum $S = \Sigma^0$. Recall the smashing theorem [B1, Corollary 4.7] (or [R]) that the smash product $S_K \wedge X$ is actually the K_*-localization of X. This implies that two CW-spectra X and Y have the same K_*-local type if and only if X is quasi S_{K*}-equivalent to Y.

In [Y2] we studied the quasi KO_*-equivalence, and moreover in [Y3] and [Y4] we determined the quasi KO_*-types of the real projective spaces RP^n and the stunted real projective spaces $RP^n/RP^m = RP^n_{m+1}$. In this note we shall be interested in the quasi S_{K*}-equivalence in advance of the quasi KO_*-equivalence. The purpose of this note is to determine the K_*-local types of the stunted real projective spaces RP^n/RP^m along the line of [Y5], in which we have already determined the K_*-local types of the real projective spaces RP^n [Y5, Theorem 3]. Our proof will be established separately in the following three cases;

i) RP^{2s+n}/RP^{2s} $(2 \le n \le \infty)$, ii) RP^{2s+2t}/RP^{2s-1} $(t \ge 1)$ and

iii) $RP^{2s+2t+1}/RP^{2s-1}$ $(0 \le t \le \infty)$.

In the proof of [Y5, Theorem 3] we first investigated the behavior of the Adams operations ψ^k_C and ψ^k_R for the real projective spaces RP^n, and then applied a powerful tool due to Bousfield [B2, 9.8] (or see [Y5, Theorem 4]). By a quite similar argument to the old case we shall determine the K_*-local types of RP^{2s+n}_{2s+1} $(2 \le n \le \infty)$ and the Spanier-Whitehead duals DRP^{2s+2t}_{2s} $(t \ge 1)$ (Theorem 2.7 and Proposition 2.8). Since two finite spectra X and Y have the same K_*-local type if and only if their duals DX and DY have the same K_*-local type [Y5, Lemma 4.7], it is easy to determine the K_*-local types of RP^{2s+2t}_{2s} $(t \ge 1)$

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