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Logarithmic Enriques surfaces, II

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Introduction

This is a sequel of our paper [2]. Every thing will be defined over the complex number field $\mathbf{C}$. Let $\bar{V}$ be a normal projective surface. A log Enriques surface can occur as the base space of a cy 3-fold with a fibration.

Definition 1. $\bar{V}$ is *a logarithmic* (*log*, for short) *Enriques surface* if the subsequent conditions are satisfied:

(1) $\bar{V}$ has at worst isolated quotient singularities;

(2) A multiple $NK_{\bar{V}}$ of a canonical divisor $K_{\bar{V}}$ of $\bar{V}$ is linearly equivalent to zero for some positive integer N;

(3) $H^1(\bar{V}, \mathcal{O}_{\bar{V}})$ vanishes.

The index of $\bar{V}$ is defined as:

$$I = \mathrm{Index}(\bar{V}) = \mathrm{Min}\{N \geq 1;\ NK_{\bar{V}} \sim 0\}.$$

A K3-surface (resp. an Enriques surface) is a log Enriques surface of index one (resp. two). It is known that $1 \leq I \leq 66$ (cf. Proposition 1.3 below). Furthermore, if I is a prime number then $I \leq 19$. Since $IK_{\bar{V}}$ is linearly equivalent to zero, there is a $\mathbf{Z}/I\mathbf{Z}$-covering $\pi\colon \bar{U} \to \bar{V}$ such that π is étale over the smooth part $\bar{V} - (\mathrm{Sing}\, \bar{V})$ of $\bar{V}$ and that $\bar{U}$ is an abelian surface or a K3-surface possibly with isolated rational double singularities (cf. [2, Definition 2.1]). In particular, the canonical divisor $K_{\bar{U}}$ of $\bar{U}$ is linearly equivalent to zero.

Definition 2. $\pi\colon \bar{U} \to \bar{V}$ is *the canonical covering of* $\bar{V}$. Actually, $\bar{V}$ determines $\bar{U}$ uniquely up to isomorphisms.

A log Enriques surface of index one is a K3-surface possibly with rational double singularities. A log Enriques surface of index 2 is an Enriques surface possibly with rational double singularities or a rational surface (cf. [2, Proposition 1.3]). The latter surfaces are classified in [2, Theorem 3.6]. Log Enriques surfaces $\bar{V}$ of index I with smooth canonical coverings $\bar{U}$ are classified in [2, Theorems 4.1 and 5.1]. In particular, if $\bar{U}$ is an abelian surface then $I = 3$ or 5.

If $\bar{V}$ has rational double singular points, we denote by $\tilde{V}$ a minimal resolution of all rational double singularities of $\bar{V}$. Then $\tilde{V}$ is a log Enriques surface of the same index as $\bar{V}$. Instead of $\bar{V}$, we can treat $\tilde{V}$ without loss of generality.

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