

**Filter-regular sequences
and
multiplicity of blow-up rings
of ideals of the principal class**

By

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1. Introduction

Let R be a graded algebra generated by finitely many elements of degree 1 over a field k and I a homogeneous ideal of R . Recently J. Herzog, B. Ulrich and this author [HTU] computed the multiplicity of the associated graded ring $\text{gr}_I(R)$, the Rees algebra $R[It]$, and the extended Rees algebra $R[It, t^{-1}]$ in terms of the degrees of the generators of I when I generated by a d -sequence of R . We had to require that the degrees of the elements of the d -sequence are non-decreasing, and we were able to give an explicit representation of the associated graded rings of these blow-up rings with respect to some refinement of the adic filtration of their maximal graded ideal, from which the multiplicity formulas followed.

In this paper we will compute the multiplicity of $\text{gr}_I(R)$, $R[It]$, and $R[It, t^{-1}]$ when I is a homogeneous ideal of the principal class, that means I is generated exactly by $\text{ht}(I)$ homogeneous elements, where $\text{ht}(I)$ is the height of I . Our main tool will be an extended version of the notion of filter-regular sequences. This notion originated from the theory of generalized Cohen-Macaulay rings [CST], and it has proven to be useful in many contexts [Br], [SV], [T2]. One can easily show that if the field k is infinite, an assumption which does not cause any problem in computing the multiplicity, then every homogeneous ideal of R can be generated by a homogeneous filter-regular sequence. For the definition and some basic properties of filter-regular sequences we refer to Section 1 of this paper. Unless otherwise specified we will denote by e the multiplicity of a given local ring with respect to the maximal ideal or of a given graded ring with respect to its (uniquely determined) maximal graded ideal.

For the associated graded rings we will use the associative formula for multiplicities to derive in Section 2 the following formula