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Propagation of analytic and Gevrey singularities for operators with non-involutive characteristics

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1. Introduction

In this paper we cosider a class of analytic operators with multiple non-involutive characteristics and study the propagation of analytic and Gevrey singularities.

To state precisely our result, we begin by recalling that $f \in C^\infty(X)$, X an open set in R^n, is said to be of Gevrey class G^d, $1 \leq d < \infty$, at $x_0 \in X$ if there exists a neighborhood V of x_0, $V \subset X$, such that

$$\sup_{x \in V} |D^\alpha f(x)| \leq C^{|\alpha|+1} \alpha!^d \tag{1.1}$$

for a constant C independent on $\alpha \in \boldsymbol{Z}_+^n$. We denote by $G^d(X)$ the space of all $f \in C^\infty(X)$ which are of class G^d at every $x_0 \in X$, and write $G_0^d(X)$ for $G^d(X) \cap C_0^\infty(X)$. $G^1(X)$ is the space of analytic functions in X.

For $d>1$, the spaces of d-ultradistributions $G^{(d)'}(X)$, $G_0^{(d)'}(X)$ are the dual spaces of $G^d(X)$ and $G_0^d(X)$ respectively and $G^{(d)'}(X)$ can be identified with the space of all elements of $G_0^{(d)'}(X)$ with compact support. We recall also that the space $D'(X)$ of all distributions in X can be identified with a subspace of $G_0^{(d)'}(X)$ for all $d>1$. If $1<d_1 \leq d$ then we have $G_0^{(d)'}(X) \subset G_0^{(d_1)'}(X)$.

The d-wave front set $\mathrm{WF}_d(f)$ of $f \in G_0^{(d)'}(X)$, $d>1$, is defined as follows: for a fixed $(x_0, \xi_0) \in T^*(X) \backslash 0$, we say that $(x_0, \xi_0) \notin \mathrm{WF}_d(f)$ if there exist $\phi \in G_0^d(X)$ with $\phi(x)=1$ in a neighborhood of x_0, and positive constants C, ε such that the Fourier transform $(\phi f)^\wedge$ of ϕf satisfies:

$$|(\phi f)^\wedge(\xi)| \leq C \exp(-\varepsilon |\xi|^{(1/d)}) \tag{1.2}$$

for all ξ in a conic neighborhood of ξ_0.

We also define the 1-wave front set (analytic wave front set) $\mathrm{WF}_1(f)$ of $f \in G_0^{(d)'}(X)$, $d>1$: let us consider a sequence $\{\phi_\nu\} \subset G_0^d(X)$, $\phi_\nu(x)=1$ in a neighborhood of x_0, such that there exists a constant C and for every $\varepsilon>0$ a consotant C_ε satisfying

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