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# Almost transversality theorem in the classical dynamical system 1

By

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## 1. Problem and main result

The purpose of this paper is to show that the Lagrange plane moving along the Hamilton flow is almost transversal to the base space, if the Hamiltonian satisfies a non-degeneracy condition.

Let $\Omega$ be a domain in $R^n$, $\tilde{\Omega}=\Omega\times R^n$ be the phase space on $\Omega$ (i.e., the cotangent bundle $T^*(\Omega)$) and $H(t, x, \xi)$ be a smooth function defined on $R\times\tilde{\Omega}$. We assume

[H.1] $\qquad \partial_x^\alpha\partial_\xi^\beta H(t, x, \xi)\in C^1(R\times\tilde{\Omega}) \quad \text{for} \quad |\alpha|+|\beta|\leq 2\,,$

We consider the Hamilton flow defined by H, i.e. the characteristic curve defined by the differential equation

$$(1.1)\qquad \frac{dX}{dt}=\frac{\partial H}{\partial\xi}(t, x, \Xi) \qquad (1.1)_s \qquad X|_{t=s}=x\in\Omega\,,$$

$$\frac{d\Xi}{dt}=-\frac{\partial H}{\partial x}(t, X, \Xi)\,, \qquad \Xi|_{t=s}=\xi\in R^n\,.$$

The solution $(X(t), \Xi(t))$ of the intial value problem $(1.1)-(1.1)_s$ exists uniquely in a maximal time interval $I_0=I_0(s, x, \xi)$, which is described as

$$(1.2)\qquad X(t)=X(t, x, \xi)=X(t, s, x, \xi)=X\,,$$

$$\Xi(t)=\Xi(t, x, \xi)=\Xi(t, s, x, \xi)=\Xi\,,$$

or

$$(1.2)'\qquad (X(t), \Xi(t))=S(t, s)(x, \xi)\,.$$

The mapping $S(t, s)$ is a local diffeomorphism in $\tilde{\Omega}$ and satisfies

$$(1.3)\qquad S(t, s)S(s, r)=S(t, r) \quad \text{(transitive law)}\,,$$

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