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On finite generation of Rees rings defined by filtrations of ideals

By

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1. Introduction

Throughout this paper we assume that all rings are commutative with unit.

Let A be a ring and $\mathfrak{F}=\{F_i\}_{i\in \boldsymbol{Z}}$ a filtration of ideals of A, i.e., $\mathfrak{F}$ is a family of ideals of A satisfying $F_i \supseteq F_{i+1}$, $F_i=A$ for $i\leq 0$, $F_i\cdot F_j\supseteq F_{i+j}$. For example, families of ideals defined below satisfy the axiom of filtrations:

- $F_i=I^i$ for an ideal I of A.
- $F_i=I^i\cdot S^{-1}A\cap A$ for an ideal I of A, where S stands for a multiplicatively closed subset of A. (When I is a prime ideal and S is equal to $A\backslash I$, F_i coincides with the i-th symbolic power $I^{(i)}$.)
- $F_i=\overline{I^i}$ for an ideal I of A, where $\overline{J}$ denotes the integral closure of an ideal J.
- $F_i=(I^i)^*$ for an ideal I of a ring A which includes a field of positive characteristic, where J^* denotes the tight closure (see [5]) of an ideal J.

We put $R(\mathfrak{F})=\sum_{i\geq 0}F_i\xi^i\subseteq A[\xi]$ (resp. $R'(\mathfrak{F})=\sum_{i\in \boldsymbol{Z}}F_i\xi^i\subseteq A[\xi, \xi^{-1}]$), where ξ is an indeterminate over A, and call it the *Rees ring* (resp. *extended Rees ring*) associated with the filtration $\mathfrak{F}$. Of course, $R(\mathfrak{F})$ or $R'(\mathfrak{F})$ is not always Noetherian even if A is Noetherian. (For example, if $(0)\neq I=F_i\subseteq \mathrm{rad}(A)$ is satisfied for $i>0$, we can prove that neither $R(\mathfrak{F})$ nor $R'(\mathfrak{F})$ is Noetherian.) In the case where $R(\mathfrak{F})$ is Noetherian, the homological properties (Cohen-Macaulayness, Gorensteinness, etc.) of such rings were studied by Goto-Nishida [4] and Viêt [11]. The purpose of this paper is to give a sufficient condition for finite generation of Rees rings associated with filtrations of ideals of Noetherian rings.

We prove the main theorem (Theorem 2.3) in the next section. Let S be a Noetherian ring and $T=S[V]$ the polynomial ring over S with a variable V. Suppose that $\mathfrak{F}=\{F_i\}_{i\in \boldsymbol{Z}}$ is a filtration of ideals of T. We put $G_i=F_i\cap S$ for $i\in \boldsymbol{Z}$. Then it is easily checked that $\mathfrak{G}=\{G_i\}_{i\in \boldsymbol{Z}}$ satisfies the axiom of

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