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Normal form of systems of partial differential and pseudo-differential operators in formal symbol classes

Dedicated to Professor Teruo IKEBE on his sixtieth birthday

By

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§ 0. Introduction

On scalar and higher order partial differential operators (or pseudo-differential operators) with characteristic roots of constant multiplicity, we know a normal form which brings a satisfactory consideration of those structures:

$$
\begin{aligned}
(0.0)\qquad p(t,x,D_t,D_x) &\equiv D_t^m+\sum_{\substack{|\alpha|\le \nu k\\ 1\le k\le m}} a_{\alpha k}(t,x)D_x^{\alpha}D_t^{m-k}\\
&= p^1(t,x,D_t,D_x)\circ\cdots\circ p^d(t,x,D_t,D_x) \mod S^{-\infty}[D_t],\\
p^j(t,x,D_t,D_x) &= (D_t-\lambda_j(t,x,D_x))^{\circ m_j}\\
&\quad+\sum_{k=1}^{m_j} b_k(t,x,D_x)\circ(D_t-\lambda_j(t,x,D_x))^{\circ(m_j-k)},\\
&\quad \mathrm{ord}\, b_k\le \nu k-1, \qquad (\nu\in \boldsymbol{N}),
\end{aligned}
$$

where $\circ$ means the operator product. (See H. Kumano-go [10] and T. Nishitani [25].) In the above, the principal part of p^j is only $(D_t-\lambda_j(t,x,D_x))^{\circ m_j}$, where $\lambda_j(t,x,\xi)$ is positively homogeneous of order ν. Using this normal form, H. Kumano-go [10] and S. Mizohata [24] characterized the C^∞ well-posedness of the Cauchy problem on $p(t,x,D_t,D_x)$. We remark that the above normal form corresponds to the following system:

$$
(0.0')\qquad \begin{pmatrix} Q^1 & & & \\ & Q^2 & & \\ & & \ddots & \\ & & & Q^d \end{pmatrix},
$$

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