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On numerical invariants of Noetherian local rings of characteristic *p*

By

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1. Introduction

Throughout this paper, all rings are commutative with identity. Let R be a Noetherian ring of characteristic p, where p is a prime number. For an ideal I of R, we denote by I^* the tight closure of I (see Definition 3.1). R is called weakly F-regular when every ideal I of R is tightly closed, that is $I^*=I$. The concept of tight closure and their fundamental properties were given by M. Hochster and C. Huneke in [4] and [5]. They proved that regular rings are weakly F-regular and that weakly F-regular rings are normal (cf. [5, § 4, § 5]).

Now, we introduce the following two invariants for a local ring $(R, \mathfrak{m})$ of characteristic p.

$$t(R):=\sup l_R(I^*/I)\,,\qquad \text{where } I \text{ runs all } \mathfrak{m}\text{-primary ideals}\,.$$

$$t_0(R):=\sup l_R(Q^*/Q)\,,\qquad \text{where } Q \text{ runs all parameter ideals of } R\,.$$

In this article, we will discuss the following:

Problems. (1) Estimate the values $t(R)$ and $t_0(R)$.

(2) When $t(R)$ (respectively $t_0(R)$) is finite, what can one say about the ring R?

Hochster and Huneck proved that $t(R)=0$ if and only if R is weakly F-regular (cf. [5, (4.16) Proposition]) and that a Gorenstein local ring with $t_0(R)=0$ is weakly F-regular (cf. [4, Theorem 5.1]). A local ring R with $t_0(R)=0$ is called F-rational (cf. [2]). They also proved that $I\subset I^*\subset \overline{I}$ and $\overline{I^n}\subset I^*$ if I is generated by n-elements (the Briançon-Skoda theorem) in [5], where $\overline{I}$ is the integral closure of I. We also introduce the following two invariants, which will be useful to inverstigate $t(R)$ and $t_0(R)$.

$$i(R):=\sup l_R(\overline{I}/I)\,,\qquad \text{where } I \text{ runs all } \mathfrak{m}\text{-primary ideals}\,.$$

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