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Global weak solutions for the equation of isothermal gas around a star

By

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1. Introduction

In this paper we study the spherically symmetric motion of isothermal gas under the gravitational force around a solid star with radius 1 and mass M. This motion is governed by the compressible Euler equation. The dynamics of an isothermal gas around the star in R^3 is written by the following system of equations.

$$
(1.1)\qquad \begin{aligned} &\rho_t+(\rho u)_r+\frac{2}{r}\rho u=0,\\ &\rho(u_t+uu_r)+p_r=-\frac{\rho M}{r^2}\ ,\\ &p=a^2\rho, \end{aligned}
$$

on $t\geq 0$ and $1\leq r<\infty$. Here, ρ and p are the density and the pressure respectively, u is the velocity normal to the surface of the star, a is a given constant and $-M/r^2$ means the gravitational force.

We investigate solutions of (1.1) which satisfy the boundary condition

$$
(1.2)\qquad u(t,1)=0\ .
$$

Let us adopt a new function $\tilde{\rho}$. Put $\tilde{\rho}=r^2\rho$. Then (1.1) becomes

$$
(1.3)\qquad \begin{aligned} &\tilde{\rho}_t+(\tilde{\rho}u)_r=0\ ,\\ &u_t+uu_r+a^2\frac{\tilde{\rho}_r}{\tilde{\rho}}=\frac{2a^2}{r}-\frac{M}{r^2}\ . \end{aligned}
$$

Next we introduce the Lagrangian mass coordinate

$$
(1.4)\qquad \tau=t\ ,\quad \xi=\int_1^r\tilde{\rho}(t,s)ds\ .
$$

Then, from (1.3) we get

$$
(1.5)\qquad \begin{aligned} &\tilde{\rho}_\tau+\tilde{\rho}^2u_\xi=0\ ,\\ &u_\tau+a^2\tilde{\rho}_\xi=\frac{2a^2}{r}-\frac{M}{r^2}\ . \end{aligned}
$$

Put $v=1/\tilde{\rho}$. Then (1.5) becomes, after changing τ to t and ξ to x,

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