J. Math. Kyoto Univ. (JMKYAZ)
34-3 (1994) 495-529

On the Picard number of Fano 3-folds with terminal singularities

To memory of Boris Moishezon

By

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Introduction

Here we continue investigations started in [N6], [N7].

Algebraic varieties we consider are defined over field $\mathbf{C}$ of complex numbers.

In this paper, we get a final result on estimating the Picard number $\rho=\dim N_1(X)$ of a Fano 3-fold X with terminal $\mathbf{Q}$-factorial singularities if X does not have small extremal rays and its Mori polyhedron does not have faces with Kodaira dimension 1 or 2. One can consider this class as a generalization of the class of Fano 3-folds with Picard number 1. There are many non-singular Fano 3-folds satisfying this condition and with Picard number 2 (see [Mo-Mu] and also [Ma]). We also think that studying the Picard number of this calss may be important for studying Fano 3-folds with Picard number 1, too (see Corollary 2 below).

Let X be a Fano 3-fold with $\mathbf{Q}$-factorial terminal singularities. Let R be an extremal ray of the Mori polyhedron $\overline{NE}(X)$ of X. We say that R has the *type* (I) (respectively (II)) if curves of R fill an irreducible divisor $D(R)$ of X and the contraction of the ray R contracts the divisor $D(R)$ to a point (respectively to a curve). An extremal ray R is called *small* if curves of this ray fill a curve on X.

A pair $\{R_1, R_2\}$ of extremal rays has the type $\mathfrak{B}_2$ if extremal rays R_1, R_2 are different, both have the type (II), and have the same divisor $D(R_1)=D(R_2)$.

We recall that a face γ of Mori polyhedron $\overline{NE}(X)$ defines a contraction f_γ: $X\to X'$ (see [Ka1] and [Sh]) such that $f(C)$ is a point for an irreducible curve C if and only if C belongs to γ. The $\dim X'$ is called the Kodaira dimension of the γ. A set $\mathcal{E}$ of extremal rays is called extremal if it is contained in a face of Mori polyhedron.

Basic Theorem. *Let X be a Fano 3-fold with terminal $\mathbf{Q}$-factorial sing-*

Communicated by Prof. M. Maruyama, March 8, 1993, Revised March 20, 1994