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The approximation of holomorphic function on Riemann surfaces

By

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Introduction

In the present paper we shall treat the approximation problem of the holomorphic function on a Riemann surface. According to Gunning and Narashimhan [1], on an arbitary open Riemann surface, there exists a locally univalent holomorphic function. The Riemann surface generated by such a function seems to be an unbranched covering surface over the complex plane. It is not easy to study such a function on the general open Riemann surface and so we shall consider a special open Riemann surface i.e. a compact Riemann surface punctured by a point. In this case a locally univalent holomorphic function has in general an essential singularity at the puncture. In the following we shall prove the approximation theorem of a holomorphic function on the punctured surface by meromorphic functions defined on the compact surface. The proof is performed, with some modifications, by the same way as Behnke and Stein did in [2]. In [2] the approximation problem has been treated exclusively at the open Riemann surface, on the contrary here the problem is concerned with the compact Riemann surface.

1. Cauchy kernel

We shall consider a compact Riemann surface R and denote its genus by g. Let $\{A_i, B_i\}_{i=1,\cdots,g}$ be a canonical homology basis and dw_i ($i=1, \cdots, g$) be the first kind of normal differentials such that $\int_{A_j} dw_i=\delta_{ij}$ and denote the third kind of normal differential by $d\Pi_{p,q}$. $d\Pi_{p,q}$ has a simple pole with residue $1(-1)$ at $p(q)$, respectively and holomorphic eleswhere and all A-periods vanish. The B-periods are given by

$$(1) \qquad \int_{B_j} d\Pi_{p,q}=2\pi i\int_{p}^{q} dw_j \quad (j=1, \cdots, g).$$

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