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Bass orders in non semisimple algebras

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0. Introduction

0. Let R be a Dedekind domain with the quotient field K. Inspired by the works of H. Bass [2], [3] for commutative rings, Drozd-Kirichenko-Roiter [8] introduced the notion of Bass orders in a finite dimensional separable K-algebra A. In this paper, we shall extend most of their local results and also the classification results of ours [9] to an arbitrary finite dimensional K-algebra A.

0.0. In the literature (cf. [8], [7], [15], [5]), hereditary orders, Bass orders and Gorenstein orders are all defined or investigated under the assumption that the ambient algebra A is semisimple or sometimes separable over K. Firstly, the definitions of these three types of orders have senses for an arbitrary A, although we have some options extending the definitions of the former two (cf. 0.2). Secondly, there are ample examples of Gorenstein orders in a non-semisimple A, for example any group algebra RG of any finite group G with the cardinality $\#G$ which is not invertible in K. Thirdly, the method to study Bass orders, especially the one adopted by Drozd-Kirichenko [7] seems in the most part free from the assumption of semisimplicity. All of these observations motivated our investigation.

As for local theory, we can do everything as well as in the semisimple case. Our main results include:
(i) Structure Theorem (4.4.1) which states that any ring indecomposable Bass order is either Morita equivalent to a primary Bass order or else Morita equivalent to one of (explicitly described) very simple Bass orders;
(ii) Classification Theorem (3.7.2) of indecomposable Λ-lattices for any Bass order Λ, withstanding the fact that Λ is no longer of finite representation type;
(iii) Classification Theorem (4.5.2) of primary Bass orders, under the same assumption as semisimple case that the residue algebra $A/\mathrm{rad}\, A$ is central over K and the residue field $R/\mathrm{rad}\, R$ is perfect with coh dim≤ 1.

All in all, the similarity in the results to the semisimple case is rather striking. More remarkably, in the above (ii), we can exlicitly determine the

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