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A Kähler structure on the punctured cotangent bundle of complex and quaternion projective spaces and its application to a geometric quantization I

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1. Introduction

As was studied in the paper [So2], the punctured cotangent bundle $T_0^*S^n$ of the sphere S^n is identified with the phase space of the Kepler problem, leading to the correspondence of the geodesic flow of the sphere to the solution curves of the problem, and it was noticed that the phase space has a complex structure. Also in the paper [Ra1] this complex structure gives a positive complex polarization, in other words, $T_0^*S^n$ has a Kähler structure whose Kähler from coincides with the symplectic form, and this structure was used to quantize the geodesic flow of the sphere([Ra3], [MT1]).

It is well-known that the geodesic flow of the sphere is periodic. In general, if we have a free $U(1)$-action generated by a positive homogeneous Hamiltonian on the punctured cotangent bundle T_0^*M of a compact manifold M, like C_l-manifold([Be]), then we have a $\mathbf{C}^*$-free action on T_0^*M and the orbit space $\mathcal{O}_M$ becomes a compact symplectic manifold with the integral symplectic form. And then, by the symplectic embedding theorem, there exists an embedding $\mathcal{O}_M \to P^N\mathbf{C}(N \gg 1)$ and T_0^*M is identified with the pull-back of the associated $\mathbf{C}^*$-principal bundle of the tautological line bundle on $P^N\mathbf{C}$. Moreover both $\mathbf{C}^*$-actions coincide. In some cases, $\mathcal{O}_M$ is seen to have a complex structure, and expected to become a Hodge manifold. So T_0^*M will have a complex structure, and it will be interesting to study whether this complex structure defines a positive complex polarization for the symplectic manifold T_0^*M. Compact symmetric spaces of rank 1 are such manifolds (of course, the sphere is mentioned above), and in these cases $\mathcal{O}_M$ is of the form $G/Z_G(T)$. Here, G denotes $SO(n)$, $SU(n)$, $Sp(n)$, or F_4, and $Z_G(T)$ is the centralizer of a certain 1-dimensional toral subgroup T in G. The space $G/Z_G(T)$ is isomorphic to the homogeneous space $G^{\mathbf{C}}/P$, where $G^{\mathbf{C}}$ is the complexification of G and P is the parabolic subgroup correspoding to $Z_G(T)$.

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