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# Remarks on the elliptic cohomology of finite groups

By

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## 1. Elliptic character

Let $Ell^*(?)$ be the elliptic cohomology based on the Weierstrass cubic

$$y^2=4x^3-g_2x-g_3$$

over

$$Ell^*=\mathbf{Z}[1/6][g_2,\ g_3,\ \Delta^{-1}](\Delta=g_2^3-27g_3^2)$$

(see [3], [11]). The coefficient ring $Ell^*=Ell^*(\mathrm{pt})$ can be viewed as the ring of modular forms on $\Gamma(1)=SL_2(\mathbf{Z})$ over $\mathbf{Z}[1/6]$. (The grading on $Ell^*$ is given by $-2\times$weight.) In other words $Ell^*$ is the ring which represents the functor

$$\{\mathbf{Z}[1/6]\text{-algebras } A\}\to\{\text{isomorphism classes of } \Gamma(1)\text{-test objects over } A\}$$

with universal test object

$$(E_{\mathrm{univ}},\ \omega_{\mathrm{univ}})=(y^2=4x^3-g_2x-g_3,\ dx/y),$$

where a $\Gamma(1)$-test object over $A$ means a pair $(E,\ \omega)$ consisisting of an elliptic curve $E/A$ and a nowhere-vanishing invariant differential $\omega$ on $E$ (see [10, Chapter II]). This identification is *natural* in the sense that the formal group law associated to $Ell$ with canonical orientation is the formal group $\widehat{E}_{\mathrm{univ}}$ associated to $E_{\mathrm{univ}}$, with parameter $T=-2x/y$.

For $n\geq 2$ let $E_{2n}\in Ell^{-4n}\otimes\mathbf{Q}$ be the Eisenstein series given by the $q$-expansion

$$E_{2n}(q)=1-(4n/B_{2n})\sum_{k\geq 1}\sigma_{2n-1}(k)q^k,$$

where

$$z/(e^z-1)=1-z/2+\sum_{n\geq 1}B_{2n}z^{2n}/(2n)!$$

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