J. Math. Kyoto Univ. (JMKYAZ)
34-4 (1994) 695-697

A remark on homomorphisms between generalized Verma modules

By

Akihiko GYOJA

1. Let $\mathfrak{g}$ be a complex simple Lie algebra, $\mathfrak{p}_s$ a parabolic subalgebra, and $\hat{\mathfrak{p}}_s$ the set of (isomorphism classes of) simple $\mathfrak{p}_s$-modules of finite dimension. For $E \in \hat{\mathfrak{p}}_s$, put $M_s(E) = U(\mathfrak{g}) \otimes_{U(\mathfrak{p}_s)} E$, where $U(-)$ denotes the enveloping algebra.

In many cases, it is known that

(A) $\dim \mathrm{Hom}_{\mathfrak{g}}(M_s(E), M_s(F)) \leq 1$ for $E, F \in \hat{\mathfrak{p}}_s$

([7], [1],$\cdots$). Based on these results, it was once conjectured that (A) is always valid. But R.S.Irving [3, 9.6] produced an example that

$$\dim \mathrm{Hom}_{\mathfrak{g}}(M_s(E), M_s(F)) = 2, \ \mathfrak{g} = D_4,$$

and this conjecture has been negatively settled. See also [4].

Recently, the author encountered a curious phenomenon. Generalizing [6], we can show a relation between
(1) irreducible factors of b-functions of semi-invariants, and
(2) intertwining operators between generalized Verma medules.
Thus it would be natural to expect that the dimensions of these Hom spaces should be related with the multiplicities of the irreducible factors of the b-functions. Curiously, it turned out that the dimension of these Hom spaces do not play any role concerning this point. To explain this phenomenon, arises the following question.

(B) Is $\mathrm{Hom}_{\mathfrak{g}}(M_s(E), M_s(F))$ irreducible in some sense?

In this note, we take up example of Irving [3] and give an affirmative answer to (B) in this case.

2. Fix a Cartan subalgebra $\mathfrak{h}$ of a complex simple Lie algebra $\mathfrak{g}$, a root basis $\{\alpha_i\}$, and a root vector X_i for each α_i. Take the root vector Y_i for the root $-\alpha_i$ so that $\alpha_i([X_i, Y_i]) = 2$. Let Γ be the stabilizer of $(\mathfrak{h}, \{\alpha_i\}, \{X_i\})$ in $\mathrm{Aut}(\mathfrak{g})$. Then $\mathrm{Aut}(\mathfrak{g})$ is a semidirect product of Γ and the inner automorphism

Received September 14, 1993
Revised June 3, 1994