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On the contraction of the Teichmüller metrics

Dedicated to Professor Fumi-Yuki Maeda on his sixtieth birthday

By

HUANG XinZhong and Masahiko TANIGUCHI

Introduction and main results

The universal Teichmüller space $T(1)$ can be represented as a quotient space of QS by the Möbius group $PSL(2,\mathbf{R})$, where QS is the group of all quasi-symmetric homeomorphisms of a circle. But QS contains another topological subgroup, which is much larger than $PSL(2,\mathbf{R})$, the subgroup S of symmetric homeomorphisms. S can be defined as the closure with respect to the quasi-symmetric topology of the group of real analytic homeomorphisms of the circle. Recently, Gardiner-Sullivan showed that $QS\mathrm{mod}S$ also have a natural complex Banach manifold structure and a natural quotient metric $\bar{d}$, which we also call the Teichmüller metric on $QS\mathrm{mod}S$, coming from the Teichmüller metric d on $T(1)$.

Since the manifold $QS\mathrm{mod}S$ is also universal in a sense (cf. [3], and also see [4]), it is important to investigate where and how extent the quotient map π contracts the metrics.

We recall some definitions. First, in $T(1)$, the Teichmüller metric can be described by using extremal quasiconformal mappings. Fix a normalized quasiconformal mapping f of the unit disk D onto itself. And denote by μ_f the complex dilatation of f. Set

$$k_f = \|\mu_f\|_\infty = \operatorname*{ess.sup}_{z\in D} |\mu_f(z)|$$

and

$$k_0(f) = \inf_g k_g ,$$

where g moves all quasiconformal mappings of D with the same boundary value as f.

We say that f is extremal (in $T(1)$-sense) if $k_f = k_0(f)$. Recall that the Teichmüller distance $d([f], [g])$, from a point $[g]$ to another point $[f]$ in $T(1)$, is equal to

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