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Local smooth solutions of the relativistic Euler equation

By

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1. Introduction

The motion of a relativistic perfect fluid in the Minkowski space-time is governed by

$$(1.1)\qquad \begin{cases} \dfrac{\partial}{\partial t}\Big(\dfrac{\rho c^2+p}{c^2-v^2}-\dfrac{p}{c^2}\Big)+\displaystyle\sum_{k=1}^{3}\dfrac{\partial}{\partial x_k}\Big(\dfrac{\rho c^2+p}{c^2-v^2}v_k\Big)=0\ , \\ \dfrac{\partial}{\partial t}\Big(\dfrac{\rho c^2+p}{c^2-v^2}v_i\Big)+\displaystyle\sum_{k=1}^{3}\dfrac{\partial}{\partial x_k}\Big(\dfrac{\rho c^2+p}{c^2-v^2}v_iv_k+p\delta_{ik}\Big)=0, \quad i=1,2,3\ . \end{cases}$$

Here c denotes the speed of light, p the pressure, (v_1,v_2,v_3) the velocity of the fiuid particle, ρ the mass-energy density of the fiuid (as measured in units of mass in a reference flame moving with the fluid particle) and $v^2=v_1{}^2+v_2{}^2+v_3{}^2$.

We assume the equation of state of the form

$$(1.2)\qquad p=a^2\rho\ ,$$

where a, the sound speed, is taken to be constant so that $0<a<c$. In particular, $a=c/\sqrt{3}$ arises in several important physical contexts. For detailed discussions of this setting, see J. Smoller and B. Temple [6].

Under the assumption (1.2), we can write the equation (1.1) as

$$(1.3)\qquad \begin{cases} \dfrac{\partial w_0}{\partial t}+\displaystyle\sum_{k=1}^{3}\dfrac{\partial w_k}{\partial x_k}=0\ , \\ \dfrac{\partial w_i}{\partial t}+\displaystyle\sum_{k=1}^{3}\dfrac{\partial f_i^k}{\partial x_k}=0, \quad i=1,2,3\ , \end{cases}$$

where

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