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The cohomology rings of $BO(n)$ and $BSO(n)$ with $\mathbf{Z}_{2m}$ coefficients

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1. Introduction

The cohomology rings of the classifying spaces for the groups $O(n)$ and $SO(n)$ with $\mathbf{Z}_2$ and $\mathbf{Z}[1/2]$ coefficients have been known for a long time, see [MS]. In 1960, E. Thomas found the group structure of $H^*(BO(n))$ with integer and $\mathbf{Z}_{2m}$ coefficients [T]. The integer cohomology ring is much more complicated so that it lasted till the year 1982 than its structure was written down in terms of generators and relations independently by E. H. Brown [B] and M. Feshbach [F]. The aim of this note is to describe the cohomology rings of $BO(n)$ and $BSO(n)$ with $\mathbf{Z}_{2m}$ coefficients in a similar way.

2. Notation and main results

Let n be a positive interger or ∞. The letters w_i and p_i will stand for the i-th Stiefel-Whitney class and the i-th Pontrjagin class of the universal vector bundle over $BSO(n)$ or $BO(n)$. The Bockstein homomorphism associated with the exact sequence $0\to\mathbf{Z}\to\mathbf{Z}\to\mathbf{Z}_2\to 0$ will be denoted δ. The mappings θ: $H^*(X,\mathbf{Z}_2)\to H^*(X,\mathbf{Z}_{2m})$ and ρ_k:$H^*(X,\mathbf{Z})\to H^*(X,\mathbf{Z}_k)$ are induced from the inclusion $\mathbf{Z}_2\to\mathbf{Z}_{2m}$ and reduction mod k, respec-tively. For a fixed $m\geq 2$, we will write only ρ instead of ρ_{2m}. For the symmetric difference of two sets I and J we will use the symbol

$$\Delta(I,J)=(I\cup J)-(I\cap J) \ .$$

Definition. *Let $\mathcal{S}_n$ be the set consisting of the elements*

z_i, x_I, y_I and u_n if n is even ,

where $i\in\mathbf{Z}$, $1\leq i<n/2$ and I ranges over all finite nonempty subsets of the positive integers less than $n/2$.

Let $\mathcal{O}_n$ be the set consisting of the elements

z_i , x_I , y_I

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