J. Math. Kyoto Univ. (JMKYAZ)
35-1 (1995) 1-33

Smooth projective varieties with the ample vector bundle $\overset{2}{\wedge} T_X$ in any characteristic

By

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In the present paper we determine the structure of smooth projective varieties with the ample vector bundle $\overset{2}{\wedge} T_X$. If X is a projective space or smooth hyperquadric, $\overset{2}{\wedge} T_X$ is an ample vector bundle. We consider the converse and obtain the following:

Main Theorem. *Let X be an n-dimensional smooth projective variety defined over an algebraically closed field whose characteristic is arbitrary. Assume that $\overset{2}{\wedge} T_X$ is ample. Then we have the following:*

1) *if $n \geq 5$, then X is isomorphic to a projective space or a hyperquadric.* (*see Theorem 6.12 and Theorem 7.11*)

2) *if the characteristic of the base field is zero and* $n \geq 3$, *then the same conclusion as in* 1) *holds.* (*see Corollary 4.5 and Theorem 5.6*).

Mori [Mo2] proved that a smooth projective variety with the ample tangent bundle is a projective space in any characteristic. Siu-Yau [S-Y] independently proved Frankel conjecture that an n-dimensional compact Kaehler manifold of positive bisectional curvature is biholomorphic to the projective space. Here we must notice that the positivity of bisectional curvature implies the ampleness of the tangent bundle over the complex number field.

An interesting problem to consider next is to determine the structure of variety with semi-ample tangent bundle. In differential geometry Mok [Mok] showed that if X is a compact complex manifold carrying a kaehler metric with non-negative bisectional curvature, then the universal covering is a product of $\boldsymbol{C}^k$, projective space and Hermitian symmetric manifold of rank ≥ 2. Here we must have in mind that the non-negative bisectional curvature implies the semi-ampleness of the tangent bundle. In this meaning it seems to us that our Main theorem is of significanse as the next step for the study of manifold with semi-ample tangent bundle.

Concerned with the subject stated above we have an attempt to determine the structure of Fano varieties by means of the quantity of rational curves of

Communicated by Prof. M. Maruyama, April 30, 1993
Revised June 9, 1994