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On holomorphic maps between Riemann surfaces which preserve *BMO*

By

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0. Introduction

We say that a nonconstant holomorphic map between plane domains is a *BMO* map if it preserves *BMO*, where *BMO* is the space of functions of bounded mean oscillation with respect to the 2-dimensional Lebesgue measure.

Reimann [16] and Jones [10] showed that *BMO* is invariant under quasiconformal maps. Hence conformal maps are *BMO* maps. Osgood [13] characterized *BMO* maps in the case of universal covering maps of plane domains. In [4] we defined *BMO* space on general Riemann surfaces and extended his result to Riemann surfaces. We also characterized *BMO* maps between plane domains in [5]. Moreover we investigated Blaschke type holomorphic maps between the extended complex planes in [6], and gave an estimate for their operator norms as *BMO* maps. In this paper we treat *BMO* maps between Riemann surfaces in succession.

In §1 we give a characterization of *BMO* maps between plane domains (Theorem 1), which extends our former results in [5]. We give also a characterization of *BMOH* maps between plane domains (Theorem 2), where *BMOH* map is a nonconstant holomorphic map which preserves harmonic *BMO* functions. In particular we show that a covering map between plane domains is a *BMO* map if and only if it is a *BMOH* map (Corollary 6).

In §2 we investigate Hahn metric on Riemann surfaces which is a generalization of the quasihyperbolic metric. We generalize several properties of the quasihyperbolic metric to Hahn metric. In particular we show that the Hahn metrical length of every closed curve which is not homotopic to a point is not less that $\pi/2$ (Proposition 9).

In §3, by using the result in §2, we investigate *BMO* maps between Riemann surfaces. In particular: (1) We give a characterization of *BMO* maps with noncompact targets (Theorem 9); (2) We give a characterization of *BMO* maps in case of covering maps (Theorem 11 and 12); (3) We give several results which indicate an essential difference between *BMO* maps with noncompact targets and *BMO* maps with compact targets (cf. Corollary 17 and 20, Theorem 14). We cannot obtain, however, a characterization of *BMO* maps with compact targets.

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