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Measure-valued branching diffusions: immigrations, excursions and limit theorems

By

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1. Introduction

Measure-valued branching diffusion processes (MBD processes) have been extensively studied concerning various problems such as ergodic behaviors [2], [17], sample path properties [4], [24], historical processes [5], [9], entrance laws [7] and so on.

In the present paper we focus upon the immigration structure of the MBD process and discuss the following problems: The first is to characterize the immigration structure associated with a given MBD process. We do this by establishing a one to one correspondence between immigration diffusion processes of the MBD process and entrance laws of its basic Markov process. The immigration process is ordinarily determined by an immigration measure supported by the state space of the basic process. However, when the basic process is an absorbing Brownian motion in a smooth domain (in this case we call the associated MBD process a super absorbing Brownian motion or simply a super ABM following Dynkin), the immigration structure consists of two parts, one is a measure supported by the interior domain and the other is a measure supported by the boundary. In particular, the latter one involves excursions of the absorbing Brownian motion from the boundary.

Secondly we discuss the immigration diffusion process of the super ABM over $(0, \infty)$, for which we derive a stochastic partial differential equation (an SPDE). When the immigration measure has compact support, so does the immigration process. We shall present a limit theorem for the range of the immigration process.

The third one is to discuss central limit theorems of immigration processes. Assuming that the basic Markov process is a Lévy process in $\mathbf{R}^d$, one can observe a "clustering-diffusive dichotomy" in the central limit theorems. More precisely, if the symmetrization of the basic process is recurrent, then the limiting Gaussian field is spatially uniform, while if the symmetrization is transient, the limiting Gaussian field is spatially fluctuating.

We remark that Dynkin [7] obtained a characterization of entrance laws of

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