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On the space of schlicht projective structures on compact Riemann surfaces with boundary

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§ 1. Introduction

Let Γ be an arbitrary Fuchsian group acting on the upper half plane $\mathbf{H} = \{z \in \mathbf{C}; \operatorname{Im} z > 0\}$. We denote by $S(\Gamma)$ the set consisting of the Schwarzian derivative S_f of all the univalent meromorphic functions f on $\mathbf{H}$ with $f \circ \gamma = \chi(\gamma) \circ f$ on $\mathbf{H}$ for some group homomorphism $\chi: \Gamma \to$ Möb. Then it turns out that $S(\Gamma)$ is a bounded closed subset of the complex Banach space $B_2(\mathbf{H}, \Gamma)$ (see § 2 for its precise definition). It is an interesting matter to investigate how (the Bers model of) the Teichmüller space $T(\Gamma)$ is embedded in $S(\Gamma)$. Generally, $\overline{T(\Gamma)} \subsetneq S(\Gamma)$ holds. In fact, first Gehring has shown that $\overline{T(1)} \subsetneq S(1)$ in [7], and later the author proved in [14] that $\overline{T(\Gamma)} \subsetneq S(\Gamma)$ for any Fuchsian group Γ of the second kind. Moreover, recently K. Matsuzaki showed in [9] the existence of certain infinitely generated Fuchsian groups Γ of the first kind such that $\overline{T(\Gamma)} \subsetneq S(\Gamma)$. But, it is still a difficult problem to decide whether $\overline{T(\Gamma)} = S(\Gamma)$ for a finitely generated Fuchsian group Γ of the first kind. (We remark that this problem is equivalent to the Bers conjecture: any b-group is a boundary group of the Teichmüller spaces.)

On the other hand, Gehring has shown in [6] that $\operatorname{Int} S(1) = T(1)$. Furthermore Žuravlev showed in [17] that $T(\Gamma)$ is the zero component of $\operatorname{Int} S(\Gamma)$ for an arbitrary Fuchsian group Γ. Thus, it is naturally conjectured that $\operatorname{Int} S(\Gamma) = T(\Gamma)$ for any Γ. In this direction, Shiga proved in [13] that the above conjecture holds if Γ is finitely generated Fuchsian group of the first kind, equivalently, if $B_2(\mathbf{H}, \Gamma)$ is finite dimensional.

The main theorem in this article (Theorem 2.1) is the claim that $\operatorname{Int} S(\Gamma) = T(\Gamma)$ for any Fuchsian group Γ uniformizing a compact (bordered) Riemann surface with nonempty boundary, in other words, for finitely generated, purely hyperbolic Fuchsian group Γ of the second kind. In order to prove this theorem, we shall utilize Gehring's method in [6] with several localization techniques for overcoming difficulties caused by the group action. Here we remark that our proof does not depend on Žuravlev's result.

The proof of the main theorem divides into several steps as follows. In § 2, we prepare terminologies and notations for later use, and state the main theorem and some lemmas. Let Γ be an arbitrary Fuchsian group, $\varphi \in \operatorname{Int} S(\Gamma)$ and f

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