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Ergodic decomposition of probability measures on the configuration space

Dedicated to Professor Takeshi Hirai on his 60th birthday

By

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Introduction

Let X be a locally compact space which satisfies the second countable axiom. Any locally finite subset of X is called a configuration in X, that is a subset $\gamma \subset X$ such that $\gamma \cap K$ is finite for any compact set $K \subset X$. Let us denote by Δ_X the space of all infinite and by B_X the space of all finite configurations in X, and set $\Gamma_X := \Delta_X \cup B_X$. We introduce a measurable structure $\mathscr{C}$ on Γ_X such that $\mathscr{C}$ is a minimal σ-algebra with which all the functions, $\gamma \in \Gamma_X \to |\gamma \cap B| \in \mathbf{R}$ are measurable, where B runs through all the Borel sets in X and $|\gamma \cap B|$ is the number of the set $\gamma \cap B$. It is known that $(\Gamma_X, \mathscr{C})$ is a standard space (See, theorem 1.2 in [3]) and hence any probability measure μ on $(\Gamma_X, \mathscr{C})$ is decomposed into conditional probability measures with resect to any sub-σ-field of $\mathscr{C}$. The subject of this paper are two kinds of measures on $(\Gamma_X, \mathscr{C})$ with well known properties and their ergodic decompositions. The first one is a $\mathrm{Diff}_0 X$-quasi-invariant probability measure μ, where X is a connected para-compact C^∞-manifold and $\mathrm{Diff}_0 X := \{\psi \mid \psi$: diffeomorphism on X with compact support$\}$. In 1975, Vershick-Gel'fand-Graev introduced elementary representations U_μ generated by these μ's and discussed fully their interesting properties in [5]. In particular they showed that U_μ is irreducible if and only if μ is ergodic. Thus our subject correspondes to an irreducible decomposition of U_μ. It will be shown in section 1 that an ergodic decomposition of $\mathrm{Diff}_0 X$-quasi-invariant probability measure is actually possible.
The second one is a consideration of Gibbs measures μ having been discussed in great detail in statistical mechanics. An ergodic decomposition of such μ relative to the tail-σ-field leads us to a remarkable fact that there exist typical extremal measures which are regarded as a base on a convex set formed by such μ's. These contents will be discussed in section 2. In both of section 1 and section 2, we denote a σ-finite non atomic Borel measure on X by m. The direct product m^n of n copies of m is naturally regarded as a measure on $\tilde{X}^n := \{(x_1, \cdots, x_n) \in X^n \mid x_i \neq x_j$ for all $i \neq j\}$ and thus an image measure $p_n m^n$ is obtained by the natural map $p_n: (x_1, \cdots, x_n) \in \tilde{X}^n \to \{x_1, \cdots, x_n\} \in B_X^n := \{\gamma \in \Gamma_X \mid |\gamma| = n\}$. We denote it by $m_{X,n}$.

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