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Asymptotic estimates for the distribution of additive functionals of Brownian motion by the Wiener-Hopf factorization method

By

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1. Introduction

In [4], we studied the supremum process of the integral of Brownian motion and obtained the following estimates: Let $b(t)$ be the one dimensional Brownian motion starting at 0. For $r>0$, $A>0$, $\sigma>0$ and $a\in\mathbf{R}$, let

$$P_{ra}(A)=P\left\{\int_0^t b(u)\,du\leqq r+at \text{ for all } 0\leq t\leq A\right\} \tag{1.1}$$

and

$$P_{ra\sigma}=P\left\{\int_0^t b(u)\,du\leq r+at+\sigma t^2 \text{ for all } 0\leq t<\infty\right\}. \tag{1.2}$$

Then it holds

$$P_{ra}(A)\sim C_1(r,a)\;A^{-1/4} \text{ as } A\uparrow\infty \tag{1.3}$$

and

$$P_{ra\sigma}\sim C_2(r,a)\,\sigma^{1/2} \text{ as } \sigma\downarrow 0 \tag{1.4}$$

with positive constants $C_i(r,a)$, $i=1, 2$, which can be given explicitly (see [4]). This is a refinement of Sinai's estimates in [9].

These asymptotics follow systematically from the theorem in [4] on a two dimensional process called the *Kolmogorov diffusion* (cf. [5]) :

$$Y(t)=y+b(t),\ X(t)=x+\int_0^t Y(u)\,du=x+yt+\int_0^t b(u)\,du. \tag{1.5}$$

Let T be the first hitting time to the positive y-axis :

$$T=\inf\{t\geq 0\ ;\ X(t)=0,\ Y(t)\geq 0\}. \tag{1.6}$$

We denote by $E_{(x,y)}$ the expectation for the diffusion starting at $(x, y)\in\mathbf{R}^2$.

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