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Note on reflection maps and self maps of $U(n)$, $Sp(n)$ and $U(2n)/Sp(n)$

Dedicated to Professor Teiichi Kobayashi on his sixtieth birthday

By

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1. Statement of result

Let $U(n)$ and $Sp(n)$ be the n-th unitary and symplectic group, respectively. We denote the complex numbers by $\mathbf{C}$, and the quaternions by $\mathbf{H}$. Let $\mathbf{F}$ be $\mathbf{C}$, $\mathbf{H}$ or $(\mathbf{C}, \mathbf{H})$. In order to describe uniformly for three cases, we write

$$G_n(\mathbf{F}) = \begin{cases} U(n) & \text{if } \mathbf{F}=\mathbf{C} \\ Sp(n) & \text{if } \mathbf{F}=\mathbf{H} \\ U(2n)/Sp(n) & \text{if } \mathbf{F}=(\mathbf{C}, \mathbf{H}). \end{cases}$$

When $\mathbf{F}$ is $\mathbf{C}$ or $\mathbf{H}$, we denote by $P(\mathbf{F}^n)$ and $Q_n(\mathbf{F})$ the projective space and the quasi-projective space, respectively. We write $Q_n(\mathbf{C}, \mathbf{H}) = \Sigma P(\mathbf{H}^n)_+$, the suspension of the union of $P(\mathbf{H}^n)$ and a point space. Recall from [1, 6, 8] (cf. §2 and §4 of this paper) that there is a map, called the *reflection map*,

$$r: Q_n(\mathbf{F}) \to G_n(\mathbf{F})$$

which induces an epimorphism on cohomology. Our result is

Theorem. *For any integer k, there exist maps $c_k : Q_n(\mathbf{F}) \to Q_n(\mathbf{F})$ and m_k: $G_n(\mathbf{F}) \to G_n(\mathbf{F})$ such that*

(1) *the following diagram commutes*

$$(1.1) \qquad \begin{array}{ccc} Q_n(\mathbf{F}) & \xrightarrow{r} & G_n(\mathbf{F}) \\ \downarrow{c_k} & & \downarrow{m_k} \\ Q_n(\mathbf{F}) & \xrightarrow{r} & G_n(\mathbf{F}); \end{array}$$

(2) *c_k induces the homomorphism of k-multiple on the integral cohomology;*

(3) *m_k induces the homomorphism of k-multiple on the ring basis of the integ-*

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