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A remark on the homotopy type of the classifying space of certain gauge groups

By

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1. Introduction

Let P_k be the principal SU (2) bundle over a closed simply connected 4-manifold X with $c_2(P_k) = k$, $\mathfrak{g}_k$ its gauge group and $\mathfrak{g}_k^0$ its based gauge group. For integers m and n, let (m, n) denote the GCD of m and n if $mn \neq 0$, $(m, 0) = (0, m) = m$. In [3], it is shown that $\mathfrak{g}_k \simeq \mathfrak{g}_{k'}$ if and only if $(12/d(X), k) = (12/d(X), k')$ where $d(X) = 1$ if the intersection form of X is even and $d(X) = 2$ if odd.

In this paper we study the homotopy type of $B\mathfrak{g}_k$. The purpose of this paper is to show the following result.

Theorem 1.1. *If $B\mathfrak{g}_k$ is homotopy equivalent to $B\mathfrak{g}_{k'}$, then $(k, p) = (k', p)$ for any prime p.*

Note that the result for $p = 2, 3$ is obtained from the result of [3].

Theorem is proved by computing the Postnikov invariant of $(B\mathfrak{g}_k)_{(p)}$ which is $B\mathfrak{g}_k$ localized at p.

By [1], we have two homotopy equivalences

$$B\mathfrak{g}_k \simeq \mathrm{Map}_k(X, BSU(2))$$

and

$$B\mathfrak{g}_k^0 \simeq \mathrm{Map}_k^*(X, BSU(2)).$$

For a fixed prime $p \geq 5$, denote $\mathbf{H}P^\infty{}_{(p)}$ by B and put (cf. [2])

$$\begin{aligned} M_{k,X} &= \mathrm{Map}_k(X, \mathbf{H}P^\infty)_{(p)} \\ &\simeq \mathrm{Map}_k(X, B) \\ M_{k,X}^* &= \mathrm{Map}_k^*(X, \mathbf{H}P^\infty)_{(p)} \\ &\simeq \mathrm{Map}_k^*(X, B). \end{aligned}$$

Consider the Postnikov invariant $\mathbf{k}^{2p-2}(M_{k,X})$. If $B\mathfrak{g}_k \simeq B\mathfrak{g}_{k'}$, then $\mathbf{k}^{2p-2}(M_{k,X}) = \mathbf{k}^{2p-2}(M_{k',X})$ for all p. So, to prove Theorem 1.1, we have only to show

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