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A remark on the homotopy type of certain gauge groups

By

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1. Introduction

Let P_k be the principal SU(2) bundle over a closed simply connected 4-manifold X with $c_2(P_k)=k$, $\mathfrak{g}_k$ its gauge group and $\mathfrak{g}_k^0$ its based gauge group consisting of bundle automorphisms of P_k which restrict to the identity on the fibre over a base point.

In [2], when $X=S^4$, it is shown that $\mathfrak{g}_k \simeq \mathfrak{g}_{k'}$ if and only if $(12, k)=(12, k')$ where $(12, k)$ is the GCD of 12 and k.

In this paper we show the similar results for closed simply connected 4-manifolds.

The homotopy type of X is determined by the intersection form Q. Define

$$d(X)=\begin{cases} 1 \text{ if } Q \text{ is even} \\ 2 \text{ if } Q \text{ is odd} \end{cases}$$

The purpose of this paper is to show following results.

Proposition 1. 1 $\mathfrak{g}_k^0 \simeq \mathfrak{g}_0^0$ *for any integer* k.

Theorem 1. 2 $\mathfrak{g}_k$ *is homotopy equivalent to* $\mathfrak{g}_{k'}$ *if and only if* $(12/d(X), k)=(12/d(X), k')$ *where* $(12/d(X), k)$ *denotes the GCD of* $12/d(X)$ *and* k *if* $k \neq 0$ *and* $12/d(X)$ *if* $k=0$.

Related results have been obtained by several authors, for example in [4].

2. Proof of Proposition 1. 1

In fact this is included in [4], essentially.

By [1], $B\mathfrak{g}_k^0 \simeq \mathrm{Map}_k^*(X, BSU(2))$. Fix $-k \in \mathrm{Map}_{-k}^*(S^4, BSU(2))$, for $f \in \mathrm{Map}_k^*(X, BSU(2))$ consider the map

$$f_{-k}: X \xrightarrow{p} X \vee S^4 \xrightarrow{f \vee -k} BSU(2) \vee BSU(2) \xrightarrow{\nabla} BSU(2)$$

where p is a pinching map and ∇ is a folding map. Then $f \to f_{-k}$ gives a

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