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Nonexistence of twisted Hecke algebras

By

Akihiko GYOJA

0. Introduction.

Let (W, S) be a Coxeter system with finite $S \neq \phi$ (cf. [1]). The group ring $\mathbf{C}[W]$ can be deformed in two ways.

(0.1) The twisted group ring. Let $\mathbf{C}[W]'$ be the vector space with the basis $\{e'_w\}_{w\in W}$. Assume that $\mathbf{C}[W]'$ has an associative $\mathbf{C}$-algebra structure, and that

$$e'_x e'_y \in \mathbf{C}^\times e'_{xy}$$

$$e'_x e'_1 = e'_1 e'_x = e'_x$$

for all $x, y \in W$. If we express $e'_x e'_y = c_{x,y} e'_{xy}$ with $c_{x,y} \in \mathbf{C}^\times$, then $c := \{c_{x,y}\}_{x,y\in W}$ becomes a 2-cocycle in $H^2(W, \mathbf{C}^\times)$. We say that the $\mathbf{C}$-algebra $\mathbf{C}[W]'$ is *obtained by twisting the group ring* $\mathbf{C}[W]$ *by the cocycle c.*

(0.2) The q-deformation of the group ring. (The Iwahori-Hecke algebra.) Let $q = \{q_w\}_{w\in W}$ be a family of non-zero complex numbers such that

(0.2.1) $$q_x q_y = q_{xy} \text{ if } l(x) + l(y) = l(xy),$$

where $l(w)$ is the length of $w \in W$. Let $H(q,W)$ be the vector space with the basis $\{T_w\}_{w\in W}$. Then there is a unique associative $\mathbf{C}$-algebra structure in $H(q,W)$ such that

(0.2.2) $$T_s T_w = \begin{cases} T_{sw} & \text{if } sw > w \\ q_s T_{sw} + (q_s - 1) T_w & \text{if } sw < w, \end{cases}$$

where $\leq$ is the Bruhat order. This $\mathbf{C}$-algebra $H(q,W)$ is called the Iwahori-Hecke algebra (cf. [1, Chap.4, §2, Ex. 23]), which we shall regard as a *q-deformation of the group ring* $\mathbf{C}[W]$.

The purpose of this note is to show that, in a sense, 'the q-deformation of the twisted group ring' does not exist.

Let us explain our result more precisely. Let notation be as above, and H a vector space over $\mathbf{C}$ with the basis $\{e_w\}_{w\in W}$.

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