J. Math. Kyoto Univ. (JMKYAZ)
36-1 (1996) 45-60

Lie algebra of the infinitesimal automorphisms on S^3 and its central extension

By

Tosiaki KORI

0. Introduction

In this paper we shall deal with a central extension of the Lie algebra of infinitesimal automorphisms on S^3. Such a central extension on the circle is famous in the name of Virasoro algebra. The Lie algebra *Vect* (S^1) of infinitesimal automorphisms on the circle is generated by (the restriction on S^1) of

$$L_m = z^m\left(z\frac{d}{dz}\right), \quad m=0, \pm 1, \cdots,$$

where we look $S^1=\{z\in C;\ |z|=1\}$, with the commutation relation

$$[L_m, L_n] = (n-m)L_{m+n}.$$

A two cocycle on *Vect* (S^1) is given by the formula

$$c(L_m, L_n) = -\frac{1}{12}n(n^2-1)\delta_{n+m,0}. \tag{0-1}$$

Virasoro algebra is the central extension associated with this two cocycle. A highest weight representation of the Virasoro algebra is generated by a highest weight representation of the affine Lie algebra S^1g (Sugawara construction) [K]. Though we have not a satisfactory theory on the highest weight representation of the (abelian) extension of S^3g [M-R] and do not know about the action of *Vect* (S^3) on the representation space of current algebra the author thinks it is worth trying to have a central extension of *Vect* (S^3).

In [K-K] it was shown that the two cocycle (0-1) is derived from the non-commutative residue on the cotangent bundle of S^1, that is,

$$c(X, Y) = \int_{|z|=1} res\ [\ln|\zeta|^2, symb\ X] \cdot symb\ Y.$$

Here *symb* X is the pseudodifferential symbol and ζ denotes fiber coordinate. (Actually their derivation of (0-1) should be corrected a little. See the

---

Communicated by Prof. T. Hirai, January 6, 1995