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Normal subgroups and heights of characters

By

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Introduction

Let G be a finite group and p a prime. Suppose we are given an irreducible character χ of G such that χ_N is irreducible for a normal subgroup N of G. Then every irreducible character ζ of G lying over χ_N is written as $\zeta = \chi\theta$ for a unique irreducible character θ of G/N. Let B (resp. $\bar{B}$) be the block of G (resp. G/N) to which ζ (resp. θ) belongs. It is natural to ask how B and $\bar{B}$ are related. If χ is the trivial character then B is just a block which dominates $\bar{B}$ and basic facts, including the relations between defect groups of these blocks, are known (cf. [11, Chapter 5, Sections 8.2 and 8.3]). (We note that we have shown, with no restrictions on N, that there exists a block of G/N dominated by B with defect group DN/N for a defect group D of B, cf. [10, Remark 4.7].) We shall show in Section 1 that, for an arbitrary χ, the situation is quite analogous to that of the usual "domination" above. The same is true when χ is an irreducible Brauer character. Actually the results are obtained in a more general setting, that is, we consider "V-domination" for suitable indecomposable G-modules V.

To explain the results in Section 2 we need to introduce some notation. Let B be a block of G which covers a block b of a normal subgroup N of G. Let ξ be an irreducible character in b. Let $T_G(b)$ be the inertial group of b in G. As in [10] we call a defect group D of B an *inertial defect group* of B if D is a defect group of the Fong-Reynolds correspondent of B in $T_G(b)$. Fix an inertial defect group D of B. Let $\mathrm{Irr}(B|\xi)$ be the set of irreducible characters in B lying over ξ. In Section 2 we shall show that

$$\min\{\mathrm{ht}(\chi) - \mathrm{ht}(\xi) \mid \chi \in \mathrm{Irr}(B|\xi)\}$$

is determined by information on DN and the $T_G(b)$-conjugates of ξ. This extends some results in [10]. As an application we shall obtain a result related to the Dade conjecture [3]. We shall also obtain a slight extension of the Gluck-Wolf theorem [5].

In Section 3 we shall give the modular version of the above.

Throughout this paper let (K, R, k) be a p-modular system. We assume that K is sufficiently large with respect to G and that k is algebraically closed.

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