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Pontrjagin rings of the Morava K-theory for finite H-spaces

By

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Introduction

In this note, we study Morava K-theory of finite homotopy associative H-spaces X with p-torsion. By using a result of Ravenel-Wilson [R-W] we compute the Pontrjagin product structure for $K(2)_*(X)$ or $K(3)_*(X)$. As its application, we give very short proofs of the Kane's theorems [K] about the relations between H-spaces with p-torsion and the exceptional Lie groups. In particular, we show that the Pontrjagin ring $K(3)_*(E_8)$ for $p=3$ is extremely simple, e.g., it is generated by only two elements as a $K(3)_*$-algebra. Moreover this $K(3)_*$-algebra structure deduces the Hopf algebra structure of the ordinary mod 3 cohomology $H^*(E_8; Z/3)$ without using any theories of classification of simple Lie algebras. These arguments are some analogue for the proof of non homotopy nilpotency of exceptional Lie groups in [R], [Y3].

§ 1. H-spaces with one even degree generator

Let X be a simply connected homotopy associative H-space. By the Borel theorem, $H^*(X, Z/p)$ is a tensor algebra of truncated polynomial and exterior algebras generated by elements of even and odd dimensional respectively. In this section we consider the case that the polynomial algebra is generated by only one element y. From Kane [K] we have $|y| = 2(p^i + p^{i-1} + \cdots + p + 1)$ for some i and $y^{p^2} = 0$. However all known examples satisfy the case $i = 1$ and $y^p = 0$. Hence we assume here

(1.1) $$H^*(X; Z/p) \cong Z/p[y]/(y^p) \otimes \Lambda\,, \qquad |y| = 2p+2$$

where Λ is an exterior algebra generated by odd degree elements. Then it is also well known (see [K]) that there are elements $x, x' \in \Lambda$ such that

$$Q_1 x = Q_0 x' = y\,, \qquad |x| = 3,\ |x'| = 2p+1,\ |y| = 2p+2$$

where Q_i is the Milnor primitive operator, i.e., $Q_0 = \beta$, $Q_1 = \beta\mathscr{P}^1 - \mathscr{P}^1\beta$.

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