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A counter-example to the q-Levi Problem in $\mathbf{P}^n$

By

Mihnea COLŢOIU

§ 0. Introduction

Let $D \subset \mathbf{P}^n$ be an open set which is locally Stein. It follows then, from the characterization of the pseudoconvexity of D by the plurisubharmonicity of $-\log \delta_D$ [10], that D is itself Stein (if $D \neq \mathbf{P}^n$). A generalization of the above statement in the q-convex case would be the following:

*) Let $D \subset \mathbf{P}^n$ be an open subset which is locally q-complete. Then D is q-convex.

We consider here the classical definitions of q-convexity as introduced by Andreotti and Grauert in [1].

The statement *) could be called the q-Levi Problem in $\mathbf{P}^n$. It is known [8] that *) has an affirmative answer if the boundary ∂D of D is smooth. In this particular case the boundary distance δ_D (with respect to the Fubini metric on $\mathbf{P}^n$) is also smooth near ∂D and $-\log \delta_D$ is a q-convex function at the points of D which are sufficiently close to ∂D.

In this paper we consider domains $D \subset \mathbf{P}^n$ with non-smooth boundary, therefore the distance δ_D is only continuous. Under the assumption that $D \subset \mathbf{P}^n$ is locally q-complete it follows then that D has certain global q-convexity properties, but with respect to some other classes of functions: D is a pseudoconvex domain of order $(n-q)$ [4], [5], D is q-complete with corners [6].

The aim of this paper is to give a counter-example to *), therefore to show that the q-Levi Problem in $\mathbf{P}^n$ does not hold.

More precisely we prove:

Theorem 1. *There exists a domain $D \subset \mathbf{P}^3$ which is locally 2-complete but D is not 2-convex.*

§ 1. The construction of the counter-example proving Theorem 1

Let us recall first some basic definitions and results which will be needed in this paper.

If U is an open subset in $\mathbf{C}^n$, a function $\varphi \in C^\infty(U, \mathbf{R})$ is called q-convex iff the Levi form $L(\varphi)$ has at least $(n-q+1)$ positive (>0) eigenvalues at any

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