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The Artin invariant of supersingular weighted Delsarte K3 surfaces

By

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1. Introduction

Let k be an algebraically closed field of positive characteristic p. Let X_k be a K3 surface defined over k. Denote by $\mathrm{NS}(X_k)$ the Néron-Severi group of X_k. It is known that $\mathrm{NS}(X_k)$ is a finitely generated abelian group with $\mathbf{Z}$-rank at most 22; put $\rho(X_k) = \mathrm{rank}_{\mathbf{Z}} \mathrm{NS}(X_k)$. As in [10], we call X_k a *supersingular K3 surface* if $\rho(X_k) = 22$. Write disc $\mathrm{NS}(X_k)$ for the determinant of the intersection matrix of $\mathrm{NS}(X_k)$. If X_k is supersingular, then

$$\mathrm{disc}\, \mathrm{NS}(X_k) = -p^{2\sigma_0(X_k)}$$

for some integer $\sigma_0 = \sigma_0(X_k)$ satisfying $1 \leq \sigma_0 \leq 10$ (cf. [1]). The integer σ_0 may be called the *Artin invariant* of X_k. In [8], Shioda showed that σ_0 takes all the 10 possible values; furthermore, in [10], he gave concrete examples of K3 surfaces for all values of σ_0 except for $\sigma_0 = 7$ and 10. In this paper, we apply Shioda's ethod (which is based on Ekedahl's algorithm of computing σ_0) to weighted Delsarte surfaces and construct supersingular K3 surfaces with Artin invariant 10.

Let $Q = (q_0, q_1, q_2, q_3)$ be a quadruplet of positive integers such that $p \nmid q_i$ $(0 \leq i \leq 3)$ and $\gcd(q_\alpha, q_\beta, q_\gamma) = 1$ for every triple $\{\alpha, \beta, \gamma\} \subset \{0, 1, 2, 3\}$. The weighted projective 3-space over k of type Q is the projective variety $\mathbf{P}_k^3(Q) := \mathrm{Proj}\, k[x_0, x_1, x_2, x_3]$ where the polynomial algebra is graded by the condition $\deg(x_i) = q_i$ $(0 \leq i \leq 3)$ (cf. [4]). Let μ_{q_i} be the group of q_i-th roots of unity in $k^\times$. Put $\boldsymbol{\mu} = \mu_{q_0} \times \mu_{q_1} \times \mu_{q_2} \times \mu_{q_3}$. Then $\boldsymbol{\mu}$ acts on $\mathbf{P}_k^3$ diagonally and we have $\mathbf{P}_k^3/\boldsymbol{\mu} \cong \mathbf{P}_k^3(Q)$ (cf. [4], §1.2.2).

Let m be a positive integer such that $p \nmid m$. Let $A = (a_{ij})$ be a 4×4 matrix of integer entries satisfying the conditions

$$\begin{cases} \text{(i)} & a_{ij} > 0 \text{ and } p \nmid a_{ij} \text{ for every } (i,j) \\ \text{(ii)} & p \nmid \det A \\ \text{(iii)} & \sum_{j=0}^{3} q_j a_{ij} = m \text{ for } 0 \leq i \leq 3 \\ \text{(iv)} & \text{given } j,\ a_{ij} = 0 \text{ for some } i. \end{cases}$$

We define a *weighted Delsarte surface in* $\mathbf{P}_k^3(Q)$ *of degree* m *with matrix* A (cf. [2], [9]) to be the surface

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