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Ropes in projective space

By

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Let C be a degree d non-degenerate integral curve in $\mathbf{P}^r$. In 1983, a remarkable paper of L. Gruson, R. Lazarsfeld, and C. Peskine showed, among other results, that C must be $(d+2-r)$-regular [18]. Such a theorem bounding the regularity in terms of d and r alone is not possible for non-reduced schemes. By considering the genus as well as the degree, Gotzmann was able to obtain bounds for the regularity of an arbitrary nonreduced one-dimensional scheme [17]. If no conditions are placed on the genus, one can construct non-degenerate locally Cohen-Macaulay schemes of degree two with arbitrarily high regularity. In general, one can construct multiplicity two structures on any curve such that the homogeneous ideal has generators in arbitrarily high degree. Multiplicity two structures on the line are called double lines and they provide us with our first example of a ribbon.

In 1986, the first author showed that double lines and their deficiency modules exhibit a form of extremal behavior with respect to liaison [27]. In 1993, M. Martin-Deschamps and D. Perrin obtained several nice bounds on the Hartshorne-Rao (or deficiency) module for an arbitrary 1-dimensional locally Cohen-Macaulay scheme [24]. Double lines exhibit extremal behavior with respect to these bounds as well. Multiplicity two structures arise naturally in questions concerning self-linkage. Rao was able to utilize this fact to obtain restrictions on the cohomology of rank two vector bundles on $\mathbf{P}^4$. Here we see that for questions concerning both regularity and liaison, relatively simple non-reduced schemes can provide us with quite interesting behavior.

Let C be a smooth and irreducible curve in $\mathbf{P}^n$ with homogeneous ideal I, and let Y be a subscheme of $\mathbf{P}^n$ with ideal J. We will call Y an *α-rope* on C if the ideal J satisfies $I^2 \subset J \subset I$ and Y is a locally Cohen-Macaulay multiplicity α structure on C. (A general definition of a rope can be found for instance in Chandler's thesis, c.f. [5], but it is straightforward to check that this general definition coincides with the one given above in the case of a smooth and irreducible curve, when everything is embedded in $\mathbf{P}^n$.) A 2-rope is called a *ribbon.*

In this paper we are interested in certain aspects of the study of ribbons and ropes. Ferrand showed that on any smooth integral curve, there exists

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