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Cohen-Macaulayness in graded rings associated to ideals

By

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1. Introduction

Let A be a Noetherian local ring with maximal ideal $\mathfrak{m}$. Let $d = \dim A$ and assume the field $A/\mathfrak{m}$ is infinite. For a given ideal I in A ($I \neq A$) we define

$$R(I) = \sum_{n \geq 0} I^n t^n \subseteq A[t] \qquad \text{and} \qquad G(I) = R(I)/IR(I)$$

(here t is an indeterminate over A) and respectively call $R(I)$ and $G(I)$ the Rees algebra and the associated graded ring of I. The purpose of this paper is to find any practical conditions under which the graded algebras $R(I)$ and $G(I)$ are Cohen-Macaulay and/or Gorenstein rings. And, because Cohen-Macaulayness and Gorensteinness in $R(I)$ are now known to be fairly determined by the corresponding ring-theoretic properties of $G(I)$ (see, for examples, [GS], [I], [TI], [GNi], [V], and [L]), in this paper we devote our attention to the problem how to check Cohen-Macaulayness or Gorensteinness in the graded rings $G(I)$. We shall develop our study along the notion, due to [HH1], *analytic deviation* $\mathrm{ad}(I)$ *of* I. Actually, for the ideals I having $\mathrm{ad}(I) \leq 2$ Huckaba and Huneke [HH1] and [HH2] have already studied Cohen-Macaulayness in graded rings $R(I)$ and $G(I)$ and the readers may consult [GNa1] and [GNa2] about Gorensteinness in them. This paper succeeds the researches [HH1], [HH2], [GNa1], and [GNa2]. Here we shall generalize their results for ideals of $\mathrm{ad}(I) \geq 3$.

To state the results precisely, we set up the following notation. Let I ($\neq A$) be an ideal in A of $\mathrm{ht}_A I = s$ and put $\lambda(I) = \dim A/\mathfrak{m} \otimes_A G(I)$, that we call the analytic spread of I. We generally have

$$s \leq \lambda(I) \leq d - \inf_{n \geq 1} \mathrm{depth}\ A/I^n$$

([B]). So the difference $\mathrm{ad}(I) = \lambda(I) - s$ is called the analytic deviation. Let J be another ideal in A. We say that J is a reduction of J if $J \subseteq I$ and $I^{n+1} = JI^n$ for all $n \gg 0$. A reduction is called minimal if it is minimal among reductions. As is well-known, a reduction J of I is minimal if and if J is generated by $\lambda(I)$ elements ([NR]). For each reduction J of I let $r_J(I) = \min\{n \geq 0 \mid I^{n+1} = JI^n\}$ and call it the reduction number of I with respect to J. We put $r(I) = \min r_J(I)$ where J runs over minimal reductions.

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