J. Math. Kyoto Univ. (JMKYAZ)
36-3 (1996) 619-640

Quasi Sure quadratic variations of two parameter smooth martingales on the Wiener space*†

By

Zong-xia LIANG

1. Introduction

Stimulated by Malliavin calculus, the theory of quasi sure analysis of Wiener functionals has been extensively developed (cf. [4, 6, 7, 8, 9, 12, 13, 16, 17, 18, 19], etc). Recently, J. Ren (cf. [17]) has studied quasi sure properties of quadratic variation of "*smooth martingales*", a notion introduced by P. Malliavin and D. Nualart [8], and his results are concentrated on studying one parameter smooth martingales and two parameter smooth strong martingales. In this paper we generalize his results in more general setting and study the quasi sure properties of two parameter smooth martingales which are not necessary strong martingale in general. This situation is much more difficulty to handle, when Malliavin calculus is involved, because the two parameter stochastic differentiation rules (cf. [1, 21, 24]) and the representation of two parameter square integrable martingales involve "*stochastic integral of the second type*", i.e., $\iint_{\Pi\times\Pi} f(\xi, \eta)dW_\xi dW_\eta$ (cf. [1]). Now let us state our results in more details.

Let N be a two parameter smooth martingale, then by [1] and [21], for each $z \in \Pi = [0, 1]^2$, N can be represented as a sum of stochastic integral of the first type and stochastic integral of the second type,

$$N_z = \int_{R_z} \phi(\eta)dW_\eta + \iint_{R_z\times R_z} \psi(\xi, \eta)dW_\xi dW_\eta$$

where W is a two parameter Wiener process and vanishing on the axes. Let $\bar{N}_z = \int_{R_z} \phi(\eta)dW_\eta$ and $M_z = \iint_{R_z\times R_z} \psi(\xi, \eta)dW_\xi dW_\eta$. It is well known that the quadratic variation processes of $\bar{N}$, M and N are given by $\langle \bar{N}\rangle_z = \int_{R_z} \phi(\eta)^2 d\eta$, $\langle M\rangle_z = \iint_{R_z\times R_z} \psi(\xi, \eta)^2 d\xi d\eta$ and $\langle N\rangle^z = \int_{R_z} \phi(\eta)^2 d\eta + \iint_{R_z\times R_z} \psi(\xi, \eta)^2 d\xi d\eta$ respectively. And by [21], we have for each z that $\langle \bar{N}, M\rangle_z = 0$. We shall prove

---

* Work supported by NSF and Doctoral Research Foundation of China
† Typeset by latex
Communicated by Prof. K. Ueno, October 11, 1995
Revised February 16, 1996