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A note on residually transcendental prolongations with uniqueness property

By

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1. Introduction

Throughout $K(x)$ is a simple transcendental extension of a field K, and v is a (Krull) valuation of K with value group G_v and residue field k_v. Let w be a valuation of $K(x)$ extending v whose residue field is a transcendental (to be abbreviated as tr.) extension of k_v; such a valuation w is called a residually transcendental prolongation of v. We say that w has uniqueness property if there exists $t \in K(x)\backslash K$ such that (i) w coincides with the Gaussian valuation v^t of the field $K(t)$ defined on $K[t]$ by $v^t\left(\sum_i a_i t^i\right) = \min_i v(a_i)$; (ii) w is the only valuation of $K(x)$ which extends v^t.

In 1990, Matignon and Ohm [3, Cor. 3.3.1, Remark 3.4] proved that if (K, v) is henselian or of rank 1, then each residually transcendental prolongation w of v to $K(x)$ has uniqueness property. Alexandru, Popescu and Zaharescu have shown that such prolongations w of v have uniqueness property provided the completion $(\hat{K}, \hat{v})$ of (K, v) is henselian and each finite simple extension of $\hat{K}$ is defectless (cf. [1, Theorem 4.5]). The converse problem is dealt with here. We prove:

Theorem 1.1. *Let v be a valuation of any rank of a field K. Each residually transcendental prolongation of v to $K(x)$ has uniqueness property if and only if the completion of (K, v) is henselian.*

2. Definition, notation and some preliminary results

Recall that for a finite extension $(K_1, v_1)/(K, v)$ of valued fields, the henselian defect is defined to be $[K_1^h : K^h]/ef$, where "h" stands for henselisation with respect to the underlying valuation and e, f for the index of ramification and the residual degree of v_1/v. We shall denote this defect by $\operatorname{def}^h (K_1, v_1)/(K, v)$ or by $\operatorname{def}^h (v_1/v)$.

The proof of the following already known lemma is omitted (cf. [4, p. 306, Lemma]).

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