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The induced homomorphism of the Bott map on K-theory

Dedicated to Professor Yasutoshi Nomura on his 60th Birthday

By

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1. Introduction

The original Bott map is a map from the unreduced suspension of a compact symmetric space into another compact symmetric space (see [5]). The complex K-theory of a compact symmetric space has been studied well (see [6], [12], [13] and [14]). The purpose of this paper is to describe the behavior of the homomorphism induced by the Bott map on complex K-theory.

Throughout this paper, G denotes a compact connected Lie group and σ an involutive automorphism of G. Then the fixed point set

$$G^\sigma = \{x \in G \mid \sigma(x) = x\}$$

of σ forms a closed subgroup of G. Let $(G^\sigma)_1$ be its identity component and H a closed subgroup of G such that $(G^\sigma)_1 \subset H \subset G^\sigma$. Then the pair (G, H) is called a compact symmetric pair, and the coset space G/H is called a compact symmetric space. If G is simply connected, then G^σ is connected, so $(G^\sigma)_1 = G^\sigma$, and G/G^σ is simply connected. Conversely, every compact, simply connected symmetric space can be expressed as a homogeneous space of a simply connected group G. When G^σ is not connected and a coset space G^σ/H is under consideration, we will use $(G^\sigma)_1$ instead of G^σ and abbreviate $(G^\sigma)_1$ to G^σ unless otherwise stated.

Associated with a symmetric space G/G^σ, there is a fibre sequence

$$G^\sigma \xrightarrow{i} G \xrightarrow{\pi} G/G^\sigma \xrightarrow{j} BG^\sigma \xrightarrow{Bi} BG$$

and a map $\xi_\sigma \colon G/G^\sigma \to G$ defined by

$$\xi_\sigma(xG^\sigma) = x\sigma(x)^{-1} \qquad \text{for } xG^\sigma \in G/G^\sigma.$$

Let $\mathbf{x} = (x_1, \ldots, x_n) \in \mathbf{K}^n$ where $\mathbf{K} = \mathbf{R}$, $\mathbf{C}$ or $\mathbf{H}$. An $n \times n$ matrix $A = (a_{ij}) \in M(n, \mathbf{K})$ with coefficients in $\mathbf{K}$ acts on $\mathbf{K}^n$ by $(A\mathbf{x})_i = \sum a_{ik}x_k$. Let $1 = I_n$ denote the $n \times n$ unit matrix and put

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