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Nonsymmetric Ornstein-Uhlenbeck semigroup as second quantized operator

By

Anna Chojnowska-MICHALIK and Beniamin GOLDYS

0. Introduction

This work deals with properties of the semigroup related to the Ornstein-Uhlenbeck operator

$$L\phi(x) = \tfrac{1}{2}\operatorname{Tr} QD^2\phi(x) + \langle Ax, D\phi(x)\rangle \tag{1}$$

in a real separable Hilbert space H. We assume that A is a generator of C_0-semigroup $S(t)$, $t \geq 0$, of bounded operators on H, Q is bounded, selfadjoint and nonnegative. By $D\phi$ we denote the Fréchet derivative of a function ϕ: $H \to \mathbf{R}$. Notice that $L\phi(x)$ is well-defined for every $x \in H$, at least for appropriately chosen cylindrical functions (see [CG1] for details). In this paper we require that

(A1a) $$\int_0^\infty \operatorname{tr} S(u)QS^*(u)\,du < \infty.$$

If (A1a) is satisfied then we can define on H the family of Gaussian measures μ_t, $t \geq 0$, and μ with the mean zero and the covariance operators

$$Q_t = \int_0^t S(u)QS^*(u)\,du$$

and

$$Q_\infty = \int_0^\infty S(u)QS^*(u)\,du$$

respectively. For simplicity of presentation we assume that

(A1b) $$\ker Q_\infty = \{0\}.$$

Let

$$R_t\phi(x) = \int_H \phi(S(t)x + y)\mu_t(dy).$$

Then the family of operators R_t, $t \geq 0$, forms a strongly continuous semigroup

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