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Euler Classes and complete intersections

Dedicated to Professor R. Sridharan on his 60th Birthday

By

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Introduction

In ([Mu2], Theorem 3.7) Murthy proved the following

Theorem 1. *Let A be a reduced affine algebra of dimension n over an algebraically closed field F with $F^nK_0(A)$ torsion free. Suppose P is a projective A-module of rank n. Let $f: P \to I$ be a surjection where $I \subseteq A$ is a local complete intersection of height n. Assume that $[A/I] = 0$ in $K_0(A)$. Then there exists a surjection from P to A. (i.e. If the top Chern class of P vanishes, then P has a unimodular element.)*

A relative version of Theorem 1 was proved by Mandal and Murthy ([MM], unpublished):

Theorem 2. *Let A be a reduced affine algebra of dimension n over an algebraically closed field F with $F^nK_0(A)$ torsion free. Let P be a projective A-module of rank n. Suppose $f: P \to I_1$ is a surjective map where $I_1 \subseteq A$ is a local complete intersection of height n. Assume that $I_2 \subseteq A$ is a local complete intersection of height n, satisfying the property that $[A/I_1] = [A/I_2]$ in $K_0(A)$. Then there exists a surjection $g: P \to I_2$.*

We note that Theorem 2 implies Theorem 1.

The theorems proved in this paper were motivated by a conjectural formulation of Theorem 1 in the case when A is a noetherian ring with $\dim A = n$. Roughly one wants to prove the following.

Conjecture. Let A be a noetherian ring with $\dim A = n$. Let P be a projective A-module with rank $P = n$. Suppose that the "n^{th} *Euler class of* P" vanishes, then P has a unimodular element.

We must of course define what one means by the n^{th} *Euler class of* P. In Section 1 we define an *Euler Class group*. The conjectural version of Theorem 2 is stated in Section 1, Question D.

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