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On the irreducible very cuspidal representations II

By

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Introduction

Let F be a non-archimedean local field and $G=\mathrm{GL}_n(F)$. Carayol [C] introduced the notion of very cuspidal representation of the maximal compact modulo center subgroup of G and showed the compact-induction of an irreducible very cuspidal representation to G is irreducible and supercuspidal. If the irreducible very cuspidal representation has an even level, it is monomial i.e. induced from a one-dimensional representation. But if the level is odd, it is not monomial and the construction of the representation is much more difficult and complicated. We remark that such phenomena occurs whenever one consider the construction of supercuspidal representation. (See e.g. [M], [B-K].)

The aim of this paper is to express the irreducible supercuspidal representation induced from a very cuspidal representation with an odd level as a $\boldsymbol{Q}$-linear combination of monomial representations. To explain more precisely, we use some notation. Let ρ be an irreducible very cuspidal representation of Z_sK_s of level N. (See Definition 1.4 and 1.7.) Then the restriction of ρ to $K_S^{[(N+1)/2]}$ contains a character ψ_u (cf. Definition 1.7.) When $N=2m$, the normalizer of ψ_u in Z_sK_s is $E^\times K_S^m$ where $E=F(u)$. Thus $\rho=\mathrm{Ind}_{F(u)^\times K_S^m}^{Z_SK_S}(\theta\cdot\psi_u)$ where θ is an appropriate quasi-character of $E^\times$. (See Proposition 1.10.) When $N=2m-1$, the normalizer of ψ_u in Z_sK_s is $E^\times K_S^{m-1}$ and the irreducible component of $\mathrm{Ind}_{K_S^m}^{E^\times K_S^{m-1}}\psi_u$ is not one-dimensional. Moreover if E/F is widly ramified, the construction of the irreducible component is not easy. In [T], the author gave the irreducible representation $\eta_{u,\theta}$ of $E^\times K_S^{m-1}$. Our main work is to calculate the character of $\eta_{u,\theta}$. Let $C=E^\times/F^\times(1+P_E)$ and $\widehat{C}$ is the character group of C. We can put

$$\mathrm{Ind}_{K_S^m}^{E^\times K_S^{m-1}}((\theta\otimes\lambda)\cdot\psi_u)=\sum_{\tau\in\hat{C}}a_{\lambda\tau}\eta_{u,\theta\otimes\tau}.$$

From the character formula of $\eta_{u,\theta}$, we can calculate the multiplicity $a_{\lambda\tau}$. Thus if we can calculate the inverse of the matrix $M=(a_{\lambda\tau})_{\lambda,\tau\in\hat{C}}$, $\eta_{u,\theta}$ is expressed as a linear combination of monomial respresentations. We can calculate the M^{-1} under some assumption (See Proposition 3.7 and Theorem 3.8.)

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