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Spectral decompositions of Berezin transformations on C^n related to the natural $U(n)$-action

Dedicated to Professor Takeshi Hirai on his 60th birthday

By

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Introduction

The Berezin transformation, which links the covariant symbol (the Berezin symbol) and the contravariant symbol (the symbol for a Toeplitz operator) of an operator A, plays an important role in Berezin's theory of quantization, see [4]. Let us begin the present paper with the definition of Berezin transformation. Consider a domain D in $\boldsymbol{C}^n$ and a Borel measure μ on D. Let $\mathfrak{H}$ be a closed subspace of $L^2(D, d\mu)$ consisting of continuous functions and we denote by P the orthogonal projection $L^2(D, d\mu) \to \mathfrak{H}$. For each $\varphi \in L^\infty(D)$ we define the Toeplitz operator $T(\varphi)$ with symbol φ by $T(\varphi)h := P(\varphi h)$ $(h \in \mathfrak{H})$. We assume that $\mathfrak{H}$ has a reproducing kernel $\kappa(z, w)$. The Berezin symbol of a bounded operator A on $\mathfrak{H}$ is the function $\sigma(A)$ on D given by

$$\sigma(A)(z) := \frac{(A\kappa(\cdot, z) \mid \kappa(\cdot, z))_{\mathfrak{H}}}{\kappa(z, z)}.$$

Then by [15, 1.19], the maps T and σ are adjoint to each other in a suitable sense. We will accordingly write σ^* for T. The *Berezin transformation* B associated to $\mathfrak{H}$ is, by definition, the positive selfadjoint operator $\sigma\sigma^*$, which turns out to be a bounded operator on $L^2(D, d\mu_0)$, where $d\mu_0 := \kappa(z, z)\, d\mu$. Moreover B is an integral operator with integral kernel given by $\dfrac{|\kappa(z, w)|^2}{\kappa(z, z)\kappa(w, w)}$, see [4] and [15].

When $\mathfrak{H}$ carries an irreducible unitary representation of a Lie group G acting on D, the operator B is G-invariant, so that it is a very interesting problem to find its spectrum. In the case where $D = \boldsymbol{C}^n$, $\mathfrak{H}$ the Fock space and G the Heisenberg group, one knows that B is expressed as the exponential of the euclidean Laplacian Δ on $\boldsymbol{C}^n$: $B = \exp(\Delta/4)$, see [4, §4], [15, 1.27] and [11, §1] etc. If D is the open unit disk $\boldsymbol{D}$ in $\boldsymbol{C}$ and if $\mathfrak{H} = \mathfrak{H}_\alpha$ $(\alpha > -1)$ is the Hilbert space of holomorphic functions on $\boldsymbol{D}$ which are square integrable rela-

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